The Grounde

OF ARTES:

Teachinge the worke and practiſe of Arithmetike, both in whole *numbres and Fractions, after* a more eaſyer and exacter ſorte then any like hath hither-to been ſette forthe.

Made by

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Doctor of Phyſik, and now of late ouerſeen and aug-mented with new and neceſſary Additions.

I. D.

All youth and Elde that reaſons Lore
Within youre breaſtes will plant to trade
Of Numbers might the endles ſtore
Fyrſt vnderſtand, than farther wade

The Bokes Verdicte.

To please or displease sure I am,
But not of one sorte, to euery man.
To please the best sorte woulde I fayne.
The frowarde displease shall I certaine.
Yet wishe I will, though not with hope,
All eares and mouthes to please or stoppe.

TO THE LOVINGE READERS.

SORE OFTEN times haue I lamented with my selfe the infortunate condition of Englande, seeynge so many greate clerkes to aryse in sundrye other partes of the worlde, and so few to appear in this our nation: where as for pregnancye of naturall witte (I thinke) few nations doe excell englishe men. But I can not impute the cause to any other thinge, than to the contempt or misregarde of learnynge. For as Englishemen are inferiour to no men in mother witte, so they passe al men in vaine pleasures to whiche they maie attaine withoute greate paine or laboure: and are as slacke to any neuer so greate cōmoditie if there hange of it any painefull study or trauelsome labore.

Howbe it, yet almen ar not of that sorte, though the moste parte bee, the more pyttie it is: but of them that are so glad not onelye with painefull study and studious payne to attaine learning, but also with as greate studie and paine to communicate their learnings

to other, & make all Englande (if it moughte bee) partakers of the same, the moste parte are suche, that vunethe theye cane supporte their owne necessarye charges, so that they are not able to beare any charges in doynge of that good, that els they desire to doe.

But a greater cause of lamentation is this, that when learned menne haue taken paines to dooe thinges for the ayde of the vnlearned, scarse they shalbe allowed for theire weldoing, but derided and scorned, and so vtterly discouraged to take in hande any lyke enterprise againe. So that if any be founde (as ther are some) that doth fauour learning and learned wittes, and can bee content to further knowledge, yea onely wyth theyre woorde, suche persons thoughe they bee rare, yet shall they encourage learned menne to enterpryse some thinges, at the leaste, that Englande may reioyce of. And I haue good hope that Englande will (after she hath take some sure taste of learning) not only bringe foorthe more fauorers of yt, but also suche lerned men, that she shall be able to compare wyth anye Realme in the worlde. But in the meane season, where so fewe regarders of learninge are, howe greatlye they are to be esteemed that doo fauoure and further it, my penne wyll not suffise at full to declare.

Therfore gentle reader, where as I doo vpon moste iuste occasion iudge, yea and knowe

know assuredly, that ther be sum mē in thys Realme whiche bothe loue, and also muche desyre to further good learninge, and yet am not wel able to wryte theyre condigne prayse for the same, I thynke it better wyth sylence to ouer passe it, than other to saye to lyttle of it, or to prouoke againste theym the malice of suche other, whyche doe nothynge them selfe that is praise worthye, and therefore canne not abide to heare the praise of any other mannes good deede. And consyderinge theyr greate fauour vnto learning, thoughe I my selfe bee not worthye to bee reckned in the noumbre of greate learned menne, yet am I bolde to put my selfe in preasse with suche abylytye as God hathe lente mee, thoughe not with so greate cunnynge as manye men, yet with as greate affection as anye manne to healpe my countrye menne, and wyll not cease daylye (as muche as my smale abilitie wyll suffer mee) to endyte some suche thynge that shall bee to the instruction, thoughe not of learned menne, yet at the leaste of the vulgare sorte, whose argument alwaies shall be suche, that it shall delite all learned wyttes, though they doe not learne anie greate thinges out of it.

But to speake of thys presente booke of Arythmetike, I dare not nor wyll not sette yt foorthe wyth anye wordes, but remitte yt to the iudgemente of all gentel

readers

readers, and namly suche as loue good learning. beseching them so to esteeme it as it dothe seeme worthy. And so other to accept the thinge for it selfe, other at the leaste to allowe my good endeuour. But I perceaue I nede not vse anye persuasions vnto theym, whose gentyle nature and fauorable minde is ready to receiue thankfullye, and interprete to the best all such interprises attempted for so good an ende, thoughe the thinge doe not alwaies satisfye mens expectation. This consydered, did bolden me to publyshe abroade this lyttle boke of the arte of numbringe, whiche if you shall receiue fauorablye, you shall encourage me to gratifye you hereafter with some greater thinge.

And as I iudge some men of so louinge a minde to their natiue contrei, that they wold much reioyce to se it to prosper in good learninge and wittye Artes, so I hope well of all the rest of Englishmen, that they will be not vnmyndefull of this due prayse, by whose meanes they are healped and furthered in any thinge. Neither oughte they to esteeme this thinge of so lyttle valewe as mannye men of lyttle discretion oftentymes doe: for who so setteth small price by the wittye deuise and knowledge of numbringe, he lyttle considereth it to be the chiefe poynte (in maner) whereby men differ from all brute beastes: for as in all other thynges (almoste) beastes

beastes are parttakers wyth vs, so in numbring we differ cleane from them, and in maner peculiarly, sith that in many things they excell vs againe.

The foxe in craftie witte exceedeth moste men,
A Dogge in smellinge hath no man his peere.
To foresighte of wether, if you looke then,
Many beastes excell man, this is cleere.
The wittinesse of Elephantes dooth letters attayne
But what cunninge doth there in the Bee remayne?
The Emmet foreseing the hardnesse of wynter,
Prouideth vitailes in time of summer.
The nightingale, the linet, the thrushe, the larke,
In Musical harmonie passe many a Clerke.
The hedgehogge of astronomy seemeth to knowe,
And stoppeth his caue where the winde wil blow.
The spider in weauinge such arte dothe shewe.
No man can him mende, nor folowe I trowe.
When a house will fall, the myse right quick,
Flee thence before, can Man do the lyke?

Mannye thinges els of the wittenesse of beastes and byrdes myghte I here saye (saue that an other tyme of theym I intende to write) wherein they excell in manner al men, as it is dayly seene: but in numbre was there neuer beast founde so cunninge, that coulde knowe or discerne one thinge from manye: as by

by daylye experience you maye well considre,
whan a Bytche hathe manye Whelpes , or a
Henne manie Chyckens : and likewaies of
other what so euer thei bee, take from them
all their yonge, lauinge onelye one, and you
shall perceiue plainely, that they misse none,
though they will resiste you in takinge them
awaye , and will secke them againe yf they
maye knowe where they bee , but elles they
will neuer misse them trulye : but take
alwaye that one that is lefte, and then will
they crie and complaine : and restore to them
that one , then are they pleased againe : so
that of numbre thys maye I iustelye saye: It
is the onely thinge (almoste) that seperateth
manne from beastes . He therefore that shall
contempne numbre , hee declareth him selfe
as brutishe as a beaste , and vnworthye to bee
counted in the felowshippe of menne . But I
truste there is no manne so foule ouerseene,
though many right smally me it regarde.
Therfore will I nowe stay to wryte againste
suche, and retourne againe to this my booke
whiche I haue wrytten in the fourme of a
Dialogue, bicause I iudge that to bee the easi-
este waye of instruction , whan the scholler
may aske euery doubt ordrelye , and the ma-
ster may answer to his question plainely.

How be it, I thinke not the contrary, but
as it is easyer to blame an other mannes
woorke then to make the lyke , so there wyll

bee

bee some that will fynde faulte, bicause I wrighte in a Dialogue: but as I coniecture, those shall bee suche as doe not, can not, either will not perceiue the reason of righte teachinge, and therfore are vnmeet to bee answered vnto, for suche men with no reason will be satisfied.

And if anye man obiecte, that other bokes haue bene written of Arithmetike alreadye so sufficientlye, that I neded not nowe to put pen to the booke, excepte I will condempne other mens writinges: to them I aunswere. That as I condemne no mans diligence, so I knowe that no one manne can satisfie euery man: and therfore lyke as many doe the esteeme greatlye other bookes, so I doubte not but some will lyke this my booke aboue any other Englyshe Arithmetike hytherto written, and namelye suche as shall lacke instructers, for whose sake I haue so plainelye set forthe the examples, as no booke (that I haue seene) hath done hitherto: whiche thinge shalbe great ease to the rude readers.

Therfore gentle reader, though this booke can be but small ayde to the learned sort, yet vnto the simple ignorant (which nedeth moste helpe) it may bee a good furtherance and meane vnto knowledge.

And though vnto the Kinge hys maiestye priuately I do it dedicate, yet I doubte not (such is his clemencye) but that hee can bee

contente, yea and muche desirous that all his louing subiectes shall take the vse of it , and employ the same to their most profit: whiche thynge if I perceiue that they thankfullye dooe , and receiue with as good will as it was written, then wil I shortly with no lesse kindnesse set forthe suche introductions into Geometrye and Cosmographye , as I haue at times promised , and as hit herto in English hath not been enterprised , wherwith I dare say, all honest heartes will be pleased , and all studious wittes greatly delyted.

I will saye no more, but lette euery man iudge as he shall se cause. And thus for this time I will staye my penne , committing you all to that true fountaine of perfecte number , whych wrought the whole world by number & measure: he is Trinitie in Unity, and Unitie in Trinitie: to whome bee all praise, honor, and glorye.

AMEN.

The contenes of the first. Dialogue.

The declaration of the profite of Arithmetike.
Numeration with an easy and large table.

Addition, Subtraction, Multiplication, Diuision. } { with dyuers examples, and some newe fourmes of working.

Reduction with diuers declarations of coynes, weightes, and measures of sundrye formes, nowe newly added to these other rules.
Progression bothe Arithmeticall and Geometricall, with certayn questions touching the same.
The Golden rule, and the Backar rule, with diuers questions therto belonging.
The double rule of proportion.
The rule of felowship, bothe with time, and without tyme.
Unto all these is added their profe.

The seconde Dialogue.

The first v. kyndes of Arrithmetik, wroughte by counters.
The common kyndes of castinge accomptes after marchauntes fashion and Auditors also.
Numbringe by the hand, newely added.

The

The contentes of the second part touching Fractions.

What a Fraction is.
Numeration in Fractions.
The order of worke in Fractions.
Multiplication.
Diuision.

Reduction of
- Diuers fractions into one denomination in 3 varieties.
- Fractions of fractions.
- Improper fractions.
- Fractions to the smallest denomination.
- Fractions into other partes of thinges.

Againe of Multiplication.
Duplation.
Againe of Diuision.
Mediation.
Addition.
Subtraction.
The Golden rule with diuers questions.
The proofe of the Golden rule.
The Backer rule.
A question of lone.
The statute of Assise of Breade and ale, recognised and applied to this time, with new tables therto annexed.
The statute of measuring of grounde, with a table and questions.

Que-

Questions of societie, with the reason of the rule, and proofes of their workes.
To finde thre numbers in any proportion.
The rule of Alligation with diuers questions, and the proofe of their workes, and many variations of suche solutions.
The rule of Falsehode or false position, with diuers questions, and their porofe.

Before the Introduction of Arithmetike, these figures muste be learned.

	1	2	3	4	5	6	7	8	9	
Figures of	i	ij	iij	iiij	v	vj	vij	viij	ix	Numbers

10	x.
20	xx.
30	xxx.
40	xl.
50	l.
60	lx.
70	lxx.
80	lxxx.
90	cx.
100	c.

C.C. two hundred. &c
D. fiue hundred. &c.
D.C. six hundred. &c.
M. thousand. &c

c. a cee, the xvi.	parte of a penny.
q. a kewe, the vii.	
q̄. a farthinge, yͤ iiii.	

ob. an halfe penny.
i d. a penny.
i s. a shillinge.
i. li, a pounde.

A DIALOGVE BE-TWENE THE MAISTER and the Scholar, teaching the arte and vse of Arithmetike with the Pen.

THE SCHOLER SPEAKETH.

SYr, suche is your auctoritye in myne estimation, that I am content to cōsent to your sayinge, & to receaue it as truth, though I see none other reason that doth leade me thervnto: wheras els in mine owne conceite it appeareth but vayn, to bestowe any time priuately in learninge of that thinge, that euery childe may and dothe learne at all times and hours, when he doth any thinge him selfe alone, and much more when he talketh or reasoneth with other.

Maist. Lo this is the fashion and chaunce of all them that seeke to defende their blinde ignorance: that when they thinke they haue made stronge reason for themselfe, than haue they proued the quite contrarye. For if numbringe bee so common (as you graunte it to bee) that no man can doo any thinge alone,

and muche lesse talke or bargain with other, but hee shall still haue to doe wth Numbre: this proueth not Numbre to bee contemptible and vile, but rather righte excellente and of highe reputation, sith it is the grounde of all mens affayres, so that without it no tale can be tolde, no communication without it can be longe continued, no bargaininge without it can duely bee ended, nor no busynesse, that man hath, iustlye completed. These commodities (if there were none other) are sufficient to approue the worthinesse of Numbre: But there are other innumerable farr passinge all these, which declare Numbre to excede all praise. Wherfore, in all great workes, are Clerkes so muche desired? Wherfore are Auditours so rychelye feed? What causeth Geometrians so highlye enhaunsed? Why are Astronomers so greatlye aduaunsed? by cause that by Numbre suche thinges they doe finde, whiche elles shoulde farre excell mans minde. Scholler. Verily sir if it be so, that these men by Numbring their cunninge doo attayne, at whose greate woorkes moste men dooe wonder, then I see well I was much deceaued, and Numbringe is a more cunninge thing then I tooke it to bee.

M. If Numbre wer so vile a thing as you did esteme it, then nede it not to be vsed so much in mens communication. Exclude Numbre and answere me to this question. How ma-

ay

ny yeares olde are you?

Schollar. Mum.

M. How many daies in a weke? how many wekes in a yeare? What landes hath youre father? How many men doth he keepe? How longe is it syth you came from him to mee?

Scholar Mum.

Maister. So that if Numbre wante, you aunswere all by Mummes: How many mile to London?

Scholar. A poke full of Plummes.

Ma. Whye, thus may you see what rule Numbre beareth, and that if Numbre be lacking, it maketh men dombe, so that to moste questions they must answere Mum.

Scholar. This is the cause sir, that I iudged it so vyle, because it is so common in talking euery while: For plenty is not deinty, as the common sayeng is.

Maister. No, nor store is no sore. perceiue you this? The more common that a thing is, beinge nedefullye required, the better is the thinge, and the more to bee desired. But in Numbringe as some of it is light and plaine, so the moste parte is difficulte, and not easy to attaine. The easier parte serueth all men in commune, and the other part requireth sum learning. Wherfore as without Numbringe a man can doo almoste nothing, so with the helpe of it, you may attayne to all thyng.

Shcolar. Yea syr? Why? then it were best

to learne the arte of Numbringe firste of all other learninge, and then a man neede learne no more, if all other come with it.

Maister. Naye not so, but if it bee firste learned, then shall a manne bee able (I meane) to learne, perceaue, and attaine to other sciences, which without it, he shoulde neuer gette.

Scho. I perceaue by your former wordes, that Astronomy and Geometry depend much of the helpe of numbringe, but that other sciences, as Musike, Physik, Lawe and Grammar, and suche lyke, haue any helpe of Arithmetike, I perceiue not.

Ma. I may perceaue your greate clerklines by the ordringe of youre sciences, but I will let that passe nowe, because it toucheth not the matter that I intende, & I will shew you how Arithmetike doth profite in al these somewhat grosselv, accordinge to youre small understanding, omitting other reasons more substantiall.

Musike. Firste (as you recken them) Musicke hath not onelye great healpe of Arithmetike, but is made and hathe hys perfectnes of it: for all Musike standeth by Numbre and Proportion.

Physik. And in physik, beside the calculation of Criticall dayes, with other thinges whiche I omitte, howe can anye man iudge the pulse rightlye that is ignoraunce of the proportion

of numbres?

And as for the lawe it is plaine, that the man that is ignoraunte of Arithmetike, is neither meete to be a Judge, neither a Proctoure. For howe canne hee well understande an other mans cause appertaining to distribution of goods, or other dettes, or of summes of monye, if hee bee ignoraunt of Arithmetike? This oftentimes causeth righte to be hindred, when the iudge either deliteth not to heare of a matter that he perceaueth not, either can not iudge it for lacke of understanding: This commeth by the ignoraunce of Arithmetike. The lawe.

Now as for grammar, me thinketh you should not doubte in what it nedeth the numbre, sithe you haue learned that Nounes of all sortes, pronounes, verbs, and participles, are distinct dyuersly by numbres: besides the varietie of nounes of numbre, and aduerbes. And if you take away numbre from Grammar, then is all the quantitie of syllables lost. And many other waies dothe numbre helpe Grammar. Wherby were all kyndes of meeters founde and made? Was it not by Numbre? Grammer

But how needfull Arithmetik is to all partes of Philophye, they may sone see that readeth either Aristotle, Plato, or any other Philosophers writinge. For all theire examples almoste, and theire probations depende of Arithmetike. It is the saiynge of Aristotle, Philosophie.

that hee that is ignoraunt of Arithmetike, is meete for no science. And Plato his maister wrote a like sentence ouer his Scoolehouse doore: Let none enter in hither (quoth hee) that is ignorant of Geometrie. Seeynge hee woulde haue all his scholers experte in Geometrie, muche rather hee woulde the same in Arithmetik, without whiche Geometrye can not stande.

Diuinitie. And howe needfull Arithmetik is to diuinitye, it appeareth, seinge so manye doctors gather so greate misteries out of Numbre, and so muche dooe write of it. And yf I shoulde go about to write all the commodities of Arithmetike in ciuille actes, as in gouernaunce of common weales in time of peace, and in due prouision and order of armies in time of warre: For numbringe of the hoste, summinge of theire wages, prouision of vitailes, vewinge of artillerye, with other armoure: Beside the cunningeste pointe of all, for castinge of grounde for encampinge of men, with such otherlike. And howe manye wayes also Arithmetike is conducible for all priuat weales, of Lordes and all possessioners, of marchauntes, and all other occupiers, and generallye, for all estates of men, besides auditors, traesorers, receyuers, stewardes, bayliffes, and such like, whose offices without Arithmetike is nothinge. If I shoulde (I say particularly repete all suche commodities

dities of this noble science of Arithmetike, it were ynough to make a very great booke.

Scho. No, no sir: you shal not neede. For I doubt not, but this that you haue sayed, were ynough to perswade anye man to thinke this arte to be right excellent and good: and so necessarye for man, that (as I thinke nowe) so muche as a man lacketh of it, so much he lacketh of his sence and witte.

Maister. What are you so farre chaunged since, by hearinge the fewe commodities in generall? By likelyhode you woulde be farre chaunged, if you knewe all the commodities particular.

S. I beseeche you sir, reserue those commodities that reste yet behinde, vnto theyre place more conuenient. And if ye will be so good as to vtter at this tyme this excellente treasure, so that I may bee somewhat enryched therby, and if euer I shalbe able, I will requite your paine.

Master. I am very glad of your request, and I will do it spedely, sith that to learne it you be so ready.

Sch. And I to your aucthorite my wit do subdue, what so euer you saye, I take it for trew.

Maist. That is to muche, and meete for no man, to be beleued in all thinges, withoute shewinge of reason. Thoughe I mighte of my Scholar some credence require, yet ex-

cept I shew a reason, I do it not desire. But nowe sith you are so earnestelye set this arte to attaine, beste it is to omitt no time, leaste some other passion coole this greate heate, and then you leaue of before you see thende.

Perseue rance in studie.

Sc. Though many there be so unconstante of minde, that flitter and turne with euerye winde, whiche often beginne and neuer come to the ende, I am none of their sorte, as I trust you partly knowe. For by my good will what I ones begin, till I haue it fully ended, I would neuer blyn.

Maister. So haue I founde you hytherto in deede, and I truste you wil increase rather then goe backe For better it were neuer to assaye, thē to shrinke and flee in the middle way. But I truste you wyll not so do, therfore tell mee briefly, what call you the science that you desire so greatly?

Scholer. Why syr, you knowe.

Mast. That maketh no matter, I woulde heare whether you knowe, and therefore I aske you. For great rebuke it were to haue studied a science, and yet can not tell how it is named.

Arithmetike.

Scho. Some call it Arsmetrike, and some Augrime.

M. And what doth those names betokē?

Sc. That yf it please you, of you woulde I learne.

Maister. Bothe names are corruptelye

written

written, Arsmetrike for Arithmetike, as the Greekes call it, and Augrim for Algorisme, as the Arabians sounde it, whiche both betoken the science of numbring. For Arithmos in Greke, is called numbre, and of it commeth Arithmetike, the arte of numbring. So that Arithmetike is a science or arte, teachinge the manner and vse of numbring. This arte may bee wroughte diuerslye, with Penne or wyth Counters. But I will firste shewe you the working with the penne, and then the other in order. ἀριθμητική. ἀριθμός.

Scho. This I will remembre. But how many things are to be learned, to attaine this arte fully?

Maist. There are recknd communlye vij. partes or woorkes of it.

Numeration, Addition, Subtraction, Multiplication, Diuision, Progression, and Extraction of rootes: to these, some men adde Duplation, Triplation, and Mediation. But as for these laste three, they are contained vnder the other seuen. For Duplation and Triplacion are contained vnder Multiplication, as it shall appeare in theyr place. And Mediation is contained vnder Diuision, as I will declare in his place also.

Sc. Yet than there remaine the firste seuen kindes of numbring.

Ma. So there dothe: Howe be it, If I shall speake exactlye of partes of numbringe,

I muste make but fiue of them : for Progression is a compounde operation of Addition, Multiplication and Diuision. And so is the extraction of Rootes. But it is no harme to name them as kindes seuerall, seyng they appeare to haue some seuerall workinge. For it forceth not so muche to contende for the numbre of them, as for the due knowledge and practising of them.

S. Than you wil that I shall name them as seuen kindes distincte. But nowe I desire you to instruct me in the vse of eche of them.

M. So will I, but it muste bee doone in order, for you maye not learne the laste as soone as the firste, but you muste learne them in that order as I did rehearse them, if you will learne them spedely and well.

Sc. Euen as you please. Than to begin, Numeration is the firste in order, what shall I doe with it?

M. First you must know what the thinge is, and than after learne the vse of the same.

NVMERATION.

Numeration is the arte to expresse and reade al summes proposed, and is of two sortes: for eyther it gathereth the value of a summe proposed, eyther else it expresseth a summe conceiued by figures and places due.

Scholer. Why? than me thinketh you put a difference betweene the value and the figures.

Maister. Yea so do I: For the value is one thinge, and the figures are an other thinge: and that commeth partly by the diuersitye of figures, but chieflye of the places wherein they be set.

S. Than I must know here three things: the Value, the Figure and the Place.

M. Euen so: but yet adde Order to them as the fourthe. And firste marke, that there are but ten figures, that are vsed in Arithmetike, and of those ten, one doeth signifye nothinge, whiche is made like an o, and is called priuatelye a Cyphar, though all the other somtime be likewise named. The other nyne are called Signifiinge figures, and bee thus figured.

A cyphar

Figures.

1. 2. 3. 4. 5. 6. 7. 8. 9.

And this is theire value.

i. ij. iij. iiij. v. vj. vij. viij. ix.

But

But here muste you marke, that euerye fygure hath two values: One alwaies certaine that it signifieth properly, which it hath of his Forme: and the other vncertaine, which hee taketh of his Place.

A place. A place is called the seate or roume that a figure standeth in. And looke howe many figures are written in one summe, so manye places hath that whole numbre. And the first place muste be called that, that is nexte to the ryght hande, and so rekeninge by order towarde the lefte hande: so that that place is laste, that is nexte to the lefte hande. As for example: If there stoode before you sixe men in a rowe, side by side, and you shoulde tell them as they stand in order, beginninge with the man that were next to your ryght hand, than he that were nexte him shoulde bee called the seconde, and so forthe to the fartheste from youre ryght hand, which is the sixt and the laste.

Sc. Sir I perceaue you wel: so mighte I recken letters or any other thynge. As if I shoulde write .viij. letters after this order, a, b, c, d, e, f, g, h. nowe muste I saye, h, is the firste, g, the ij, f the iij, e the iiij, d the v, c the vj, b the seuenth, and a the eight.

M. That is well done. And after the same sorte vse hereafter, that what I declare by one example, doo you expresse by an other, and so I shall perceaue whether you vnder-

stande

stande it or no. And so passe ouer nothinge, tyll you perceaue it well, and bee experte therein.

Sc. Syr, I pray you howe many of these places be ther in all?

M. There is no certaine numbre of them, but they are somtimes more, and somtimes fewer, according to the sum that is expressed. For so many as the figures are, so manye are the places: and the laste place is so called, not bicause it is last of all other, but it is the laste of that presente summe, and it maye bee the middle place in an other summe.

Sc. Me seemeth I perceaue this verye well, as touchinge the ordre of reckenynge of the places; But as for the numbre of them, you say there is no certaintye. Nowe there resteth to declare the value of the figures by diuersitye of places, whiche you called The value vncertaine.

Value vncertaine.

Maist. But first let me heare whether you knowe perfitely the certeyn value.

Sc. Yes syr as you wrote them, so I marked them.

Mayster. How write you then. v?

Scholer. By this figure. 5.

Maister. And how. vj.

Scholar. Thus. 6.

M. Write these three numbres eche by it selfe as I spake them. vij. iiij. iij.

Scholar. 7. 4. 3.

M.

Maist. Howe write you these four other, ij, x, ix, viij?

S. Thus (I trowe) 2, 1, 6, 8.

M. Nay there you misse: Looke on mine example againe.

Scho. Syr troth it is, I was to blame, I tooke 6 for 9, but I will be warer hereafter.

M. Nowe then take heede, these certaine values euerye figure representeth, when it is alone written without other figures ioyned to him. And also when it is in the first place, though many other doo folow: as for exaumple. This figure 9 is ix. standing now alone.

Sch. How, is hee alone and standeth in the middle of so many letters?

M. The letters are none of his felowes. And if you were in Fraunce in the middle of M. Frenche men, if there were none Englishe man with you, you woulde recken your selfe to bee alone.

Scho. So it is. Then 9 without more figures of Arithmetik, betokeneth ix, whatsoeuer other letters be about it.

M. Euen so, and so doth it, if it bee in the first place ioyned with other, howe manye so euer do folowe, as in this example. 3 6 7 9. you see 9 in the first place, and doth betoken nyne, as if he were alone.

Sc. I perceiue that. And dooth not 7 that standeth in the seconde place, betoken. vij? and. 6. in the third place betoken. vj? And so

3 in the fourth place, betoken thre?

M. Their places be as you haue sayd, but their values are not so. For as in the firste place euery figure betokeneth his own value certain only, so in the secōd place euery figure betokeneth his own value certain x. times: as in the example, 7 in the secōde place is vij. times x. ẏ is, lxx. And in the third place, euery figure betokeneth his own valew a hundreth times, so that 6 in ẏ place betokeneth. vj. C. And in the iiij. place euery figure betokeneth his owne valewe a M. times, as in the foresaide numbre 3. in the fourth place standeth for iij. M. And in the fifth place, euerye figure standeth for his owne value. x. M. times. And in the vj. place, a C. M. times. And in the. vij. place, a M. M. times. And in the viij. place, x. M. M. times: so that euery place exceedeth the former ten times.

Scho. As thus, if I make this Numbre at all aduẽtures, 9 1 3 5 9 6 8 4, here are viij places. In the first place is 4, and betokeneth but foure: in the seconde place is 8, and betokeneth x. times eight, that is, lxxx. In the thirde place is 6, and betokeneth, vi. C. In the fourthe place 9. is ix M. And 5 in the v. place, is x. M. times 5, that is. l. M. So 3 in the sixte place, is a C M. times 3. that is CCC M. Then one in the vij. place, a M M. And 9 in the viij place, x. M M times 9, that is. xc. M M. But nowe I can not

easily

easily nor quickly reade it in order.

M. That shal you practise by this meanes. Firste put a pricke ouer the fourthe figure, & so ouer the. vij. And if you adde so many ouer the x, xiij, xvi, and so forthe, still leauinge two figures betwene eche two pricks. And those roomes betweene the prickes are called Ternaries.

Ternaries. Trinities.

Then begin at the last prick, and see how manye figures are betweene him and the end, whiche can not passe three, reckeninge hymselfe for one: then pronounce them as if they weare written alone from the reste, and adde at the end of theire value so many times thousand as your numbre hath prickes.

After that come to the nexte thre figures, and sound them as if they were a parte from the rest, and adde to theire valewe so manye times thousandes, as there are prickes betweene them and the first place of your whole nūbre. And so do by euerye other. iij. figures folowinge, if you haue moe. As in example, 9 1 3 5 9 6 8 4. this was youre numbre.

Put a prick ouer 9 in the fourth place, and ouer one in the vij. place, and then no more, (for your places come not to tenne) as thus,

$$9\dot{1}35\dot{9}684.$$

Now go to the laste prick ouer 1, and take it, and the figure 9 that foloweth it, and valewe them alone.

Scholar. 9 1, that is. xci.

M.

M. So is it: but than adde for the Numbre of your prickes twise M.

Scholer. That is xci. thousand thousand.

M. So is it. Then take the three other figures from 1 to the nexte pricke, and valew them.

Scholer. 3 5 9. that is CCC. lix.

M. Nowe adde for the one pricke that is betwene them and the first place, M.

Scholer. CCClix. thousande.

M. Then come to the other three figures that remaine.

Scholer. 6 8 4. that is, vj C. lxxxiiij.

Maister. Nowe haue you valewed all. And at ỹ end of this last nūbre you shal ad nothing, because there remaineth no prick nor nū bre after it: yet proue in an other Numbre, as thus. 2 3 0 8 6 4 0 8 9 1 0 5 3 4 0.

Schol. 2 3 0 8 6 4 0 8 9 1 0 5 3 4 0. I haue pricked them as you taught mee, but I am in doubte, whether I haue done well or no, because of the Ciphers: For I remembre, you tolde me that they do signify nothing, and therefore I doubt whether I shold recke the for a figure in setting of the pricks: and againe I knowe not wherefore they serue.

Ma. That will I tell you now. In deede they are of no valewe theim selfe, but they serue to make vp numbre of places, and so maketh the figure folowinge them to bee in

a further place, and therefore to signifye the more value: as in this example, 9 0, ẏ cipher is of no value, but yet hee occupieth the firste palce, and causeth 9 to be in the second place, and so to signify .x. times 9, that is, xc. so that two cyphers thrusteth the figure folowinge them into the thirde place, and so forth.

S. Than I perceaue, in the example aboue, I haue pricked well inough: for thoughe that cyphar that is pricked signifie nothinge, yet muste he haue the pricke, bycause hee came in the xiij. place. Than wyll I proue to nombre that summe. Fyrst there is 2 3 0 M, M, M, M. and then foloweth 8 6 4 M, M, M. And what shall I nowe do? There is a cyphar in the thirde place, and no figure after hym, but they that I haue reckned.

Ma. He dyd serue for them that you haue all ready reckened, to make them in a place further then they sholde be, if he were away: and therfore now you shall let him go. And so do alwaies when he occupieth the place nexte before any pricke, whiche is the laste of that Ternary: and a cyphar in the laste place doth nothing.

S. Thē I shall say but 8 9 thousand M.

Maister. No, but go forth.

S. 1 0 5 thousande. Now are all prickes spent, and yet remaine 3 4 0, so that I must valewe them CCC.xl. only.

M. Now can you recken after this sorte

An

And remembre that euery suche roume so parted is, called a Ternary or Trinitie. Trinitie.

Some do parte such greate numbres with letters, after this manner:

c b a c b a c b a c b a c b a

2 3 0 8 6 4 0 8 9 1 0 5 3 4 0. In whiche example ye maie se, that a supplieth the roume of your pricke. And some do parte the numbres with lynes after this forme.

2 3 0|8 6 4|0 8 9|1 0 5|3 4 0. wher you see as many lines as you made prickes, to one intent, saue ŷ the lines doth more plainly parte euery three figures according as they should be valewed vnder one denomination.

S. Yea syr, but yf you shoulde shewe mee a numbre so parted, I should take it for many numbres, and not for one.

M. So myght you doo, not knowinge my meaning. But what if I did set forth the nōbre without lynes, and your selfe (for the ease of reckening) dyd so parte it with lines, wold you forget wherefore ye dyd it, and then take them for many numbres?

S. No I trow not, but yet I doubte.

M. Then vse that that you lyke beste, for all the thre waies are to one entent, saue (as I said) that the lynes dothe more plainelye dystinct the Denominacions.

Scholar. What call you Denominacions? Denomination.

Maist. It is the laste valewe or name added to any summe. As when I saye; CCxxij. pounds

poundes: pounds is the denomination. And like waies in saing: 25 men, men is the denomination, & so of other. But in this place (that I spake of before) the laste numbre of euery Ternary, is the denomination of it. As of the fyrst ternary, the denomination is unitees: and of the seconde ternary, the denomination is thousandes: and of the thirde ternary, thousand thousandes, or millions: of the iiii, thousand thousand thousandes, or thousand millions: and so forth.

S. And what shall I call the valeu of the iii figures that may be pronouced before the denominators? as in sayng, 203000000, that is CC iii millions. I perceiue by youre wordes, that millions is the denomination, but what shall I call the CC iii. ioyned before the millions?

Numerator. Valower Summe.

M. That is called the Numerator or valewer, and the whole summe that resulteth of them bothe, is calcd the Summe, Valeue or Numbre.

S. Now is there any thyng else to be learned in Numeration? or else haue I learned yt fully?

Maist. I might here shewe you who were the first inuentors of this arte, & the reasõs of all these things that I haue taught you, but that will I reserue tyl yee haue learned ouer all the practise of thys arte, leaste I shoulde trouble your witte with ouer many

ny thinges at the first.

But yet this muste you marke, that there are three kindes of numbre: one called dygets; an other articles: and the thirde myxte numbres. A diget is anye numbre vnder 10 as this, 1, 2, 3, 4, 5, 6, 7, 8, 9. And 10 with all other that may be deuided into x. parts iust and nothing remayne, are called Articles, such are 10, 20, 30, 40, 50, &c. 100, 200, &c. 1000, &c. And ꝥ nũbre is caled mixt, ꝥ containeth articles, or at the leste one article & a dygete: as 12, 16, 19, 2138, 107, 1005. & so forth. And for ꝥ more ease of vnderstãdig & remembrãce, marke this: The diget numbre is neuer written with more then one figure, but the article and the myxt numbre are euer written with more then one figure. And thus they differ, ꝥ the article hath euermore this cypher 0, in his firste place: and the myxte numbre hath euer there some digitte.

The kyndes of nũbre.

Digites.

Articles

Mixte.

Scho. By these laste wordes, I perceaue it much better then I dyd before, and nowe (I thinke) I wyll neuer misse to knowe those three a sunder.

M. If you remembre nowe all that I haue sayde, you haue learned sufficiently this firste kynde of Arithmetike, called Numeration. Howbeit, I wyll yet exhorte you now, to remembre bothe this that I haue sayde, and all that I shall say, and to exercyse your selfe in

the practise of it: for rules withoute practise are but a lyghte knowledge: and practise it is, that makethe men perfecte and prompte in all thinges.
And as youe haue learned to gather the valewe of a summe proposed, so muste you practise to expresse anye summe with their due figures, as for a profe: How expresse you this summe: Fiue thousande, two hundred, fiftie and seuen?

Sc. This troubleth me now, whether I shoulde beginne at the firste figure, or at the laste. For reasone (mee thynkethe) shoulde cause me to beginne at the fyrste: and yet yf I write it as you speake it, I muste begin at the laste.

Mast. When you knowe your places perfectly, you may beginne where you lyste. But the more ease for youre hande is to beginne withe the laste, that is to say, as I dyd speak them. Yet for the more suretye, a while you may beginne with the fyrste, repetinge my wordes backwarde, thus: Seuen, fyfty, two hundred, fyve thousande: or else soundinge them all by their digitte or valower, as thus: Seuen, fiue, twoe, fiue: for that waye is easiest. But then must you looke well, whether ther be any cyphar in your summe, that he maye bee sette in his place. As yf youre laste valewer of youre summe (as you speake it) bee aboue 9, then is there a cyphar in

the

the fyrste place, And if it be a hundred or aboue, than is there twoo cyphars, one in the fyrste place, and an other in the seconde. and so foorthe.

But bycause thys thynge is suche, that canne not be sette foorthe withoute manye wordes, I thinke beste here now at the ende of Numeration, to adde a table, easye and ready for the fyrste exercise of it.

Lo, this is that table.

The right syde or hande.

The names of the digeites, valewes certaine or valowers.

The denominatours of the places, or valewe vncertaine.	Nyne.	Eyght.	Seuen.	Sixe.	Fyue.	Foure.	Three.	Twoo.	One.	Ciphar.	The order of the places.
Vnities.	9	8	7	6	5	4	3	2	1	0	Fyrste.
Tennes.	9	8	7	6	5	4	3	2	1	0	Second.
Hundredes.	9	8	7	6	5	4	3	2	1	0	Thyrde.
Thousandes.	9	8	7	6	5	4	3	2	1	0	Fourth.
x. Thousandes.	9	8	7	6	5	4	3	2	1	0	Fyfte.
Millions.	9	8	7	6	5	4	3	2	1	0	Syxte.
C. of thousãds.	9	8	7	6	5	4	3	2	1	0	Seuenth.
x. of millions.	9	8	7	6	5	4	3	2	1	0	Eightet.
C. of millions.	9	8	7	6	5	4	3	2	1	0	Nynthe.
M. of millions.	9	8	7	6	5	4	3	2	1	0	Tenthe.
x. M of milliõs.	9	8	7	6	5	4	3	2	1	0	Eleuenth

The lefte syde or hande.

This table (as you may see) hathe aleuen places, and in eche of them are set all the digetes, whose certaine valewe is written on the righte hande of the table, and the valewe vncertaine on the lefte hande. So that by this

[t]his table you may learne bothe howe to ex[p]resse any numbre that you lyste, (yf that it [ex]cede not .xj. places) that is to saye, lxxx. thousande millions, and so may you by the healpe of it valew all numbres proposed vnder [t]he sayd numbre.

For example: Take the summe that I proposed before, which was, fiue thousand, two [h]undred, fyftye and seuen. And if you will [e]xpresse it, take ÿ first numbre (as I speake it) [w]hich is fiue M, whose valower or certaine [v]alew is v, and his vncertaine valewe or de[n]omination is M. Firste you shall seeke at [t]he right hande the valower, v. Than seeke [a]longe vnder the title of denomination to[w]ard the lefte hande, till ye finde thousandes, [a]nd vnder it righte, at the foote of the table, [i]s the numbre of the place that is the fourth, [w]herein you muste write youre digitte or va[l]ower, fyue.

Afterwarde come to the seconde part of the [n]umbre, two hundred, whose valower is 2, [a]nd his denomination C. Seeke two at the [r]ighte hande of the table, and goe alonge vn[d]er the denominations, towarde the lefte hande, [t]yl you come vnder C, than looke to the foote [o]f the table, and there shal you see the numbre [o]f the place, that is to say, iij, wherein you [m]ust set your digette 2.

[T]han doe so by youre other two numbres [t]hat remaine, and you shall finde fiue in

 the

the seconde place for your fyfty, and 7 in the first place for your seuen. And thus may you do with other numbres.

S. Maister I thanke you hartely. I perceiue you seeke to instruct me moste plainly and briefly, and not to hyde youre knowledg in subtile wordes as manie do. For this rule is so plaine, that I can desire it no playner. And though it seeme somwhat longe, yet I perceaue it to be a sure waie.

M. So is it, and though it be longe, yet it is neither to longe, neyther to playne for yonge learners that lacke practise: for thys table is in steede of a teacher, to theyme that lacke one. But nowe I truste I haue sayd ynough of Numeration: whiche after you haue well practysed, then maye you learne foorthe.

Why numbres are placed backewarde.

S. Yet I pray you in one thynge to tell me your iudgment. Why do men recken the ordre of the places backewarde, fro the right hande to the left?

M. In that thinge all men do agree, that the Chaldays, whiche fyrste inuented thys arte, did these figures as they set all their letters. for they wryte backwarde as you tearme it, and so doo they reade. And that may appeare in all Hebrewe, Chaldaye and Arabike bookes, for they be not onely written from the ryght hande to the lefte, and so muste be readde, but also the ryght end

of the booke is the beginninge of yt : where as the Greekes, Latines and all nations of Europe, doe write and reade from the lefte hande towarde the right. And all theyr bokes begyn at the lefte syde.

S. That reason doth satisfie me.

M. It nother satisfieth me, nother liketh mee well, bicause I see that the Caldays and Hebrues doe not so vse their own numbres, as at an other time I will declare. But thys playne reason maye beste satisfie you presentlye. That seinge in pronouncinge of numbres wee keepe the ordre of oure owne readinge, from the lefte hande to the righte: And againe, wee doe euer name the greater numbres before the smaller, it was reason that the lesser places containinge the lesser numbres, shoulde bee sette on the righte hand and the greater places containinge the greater numbres, to procede towarde the lefte hande.

S. This reason to mee is so playne, that it seemeth now againste reason to make a doubt of that ordre. So that nowe for Numeration, I am clearlye satisfied: so that onlye practise shall make me fullye readye and experte in it. And in the meane season I desyer to learne the other kyndes of Arithmetyke.

M. That is well sayd: but what shoulde you nexte learne, can you tell?

S. I remembre you sayde that Addition

was

was next.

Maister. Euen so: and what that is, must you firste knowe.

ADDITION.

Addition is the reduction and bringinge of two summes or more into one: as if I haue 160 bookes in the Latine tongue, and 136 in the Greeke tongue, and would knowe howe manye they be in all. I must wryte those numbres, one ouer an other, writing the greatest numbre highest, so that the firste figure of the one, bee vnder the first figure of the other. And the seconde vnder the seconde, and so forthe in ordre.

When you haue so done, draw vnder them a ryghte line, then will they stande thus.

$$\begin{array}{r} 160 \\ 136 \\ \hline \end{array}$$

Now begin at the firste places towarde the right hande alwaies, & put together the .ij. first figures of these two summes, and loke what commethe of them, write vnder them, right vnder the line. As in saying, 6 and 0 is 6. Wryte 6 vnder 6: as thus.

$$\begin{array}{r} 160 \\ 136 \\ \hline 6 \end{array}$$

And then go to the seconde figures, and doe likewise: as in sayyng, 3 and 6 is 9. write 9 vnder 6 and 3, as here you see.

$$\begin{array}{r} 160 \\ 136 \\ \hline 96 \end{array}$$

And

And lyke waies do you with the figures that be in the thirde place, saying: 1 and 1 bee 2: write 2 under them, & then will your whole sum appeare thus.

1	6	0
1	3	6
2	9	6

So that nowe you see, that 1 6 0, and 1 3 6 make in all, 2 9 6.

S. What? this is very easy to do, me thinketh I can do it euen syth.

There came through Cheapeside two droues of cattel, in ye firste was 8 4 8 sheepe, and in the seconde was 1 8 6. other beastes.

Those two summes I must write as you taught me, thus.

8	4	8
1	8	6

Than if I putte the two first fygures togither, saying: 6 and 8 they make 1 4. That must I writte under 6 and 8, thus.

8	4	8
1	8	6
	1	4

M. Not so. and here are you twise deceaued. Fyrst, in goyng about to adde togither two summes of sundry thinges, whiche you ought not to doo, except you seeke onely the numbre of theme, & care not for the thynges. For the summe that should result of that addition, should bee a summe nother of sheepe nor other beastes, but a confused summe of bothe.

Howe be it sometimes ye shall haue summes of dyuers denominations to be added, of which I wyll tell you anone. but fyrste

I

I will shewe you, where you were deceiued in an other pointe, and that was in writyng 14, which cam of 6 and 8, vnder 6 and 8, which is vnpossible: For howe can two fygures of two places bee written vnder one fygure and one place?

Scholar. Truthe it is: but yet I dyd so vnderstand you.

M. I saide in deede that you shoulde wryte that vnder them, that dyd resulte of theym bothe together: whiche sayinge is alwaies trew, yf that summe be not excede a Diget. But if it bee a mixte numbre, then muste you write the diget of it vnder youre figures, as I haue saide before: but and yf yt be an Article, then write 0 vnder them, and in bothe sortes you shall keepe the Article in youre mynde, And therefore when you haue added your second figures, which occupie the place of tennes, you shall put that 1 therto which you kept in your minde, for though it were ten in dede, yet in that place it is but as one, bycause that euerye 1 of that place is tenne for it is the place of tennes. And in like manner: yf you haue in the seconde place so great a numbre, that it amounteth aboue 9, than write the digetes, and reserue the article in youre minde, euer adding it to the nexte place folowinge, and so of all other places, howe manye so euer you haue. And yf you haue Myxte numbre, when you haue added your

[sa]me figures, then wryte the digite vnder the same figures, and the article in the nexte place beyonde them: so shall youre numbre resulting of Addition, haue one place more than the numbres whiche you shoulde adde togytheer.

S. Now doe I perceaue you, & the reason of this is, (as I vnderstand) bicause that no one place cane contaiue aboue 9, whiche is the greatest fygure that is, and then al tennes or articles muste be put to the nexte place folowing. for euery place (as I may see) exceadeth the other place next before him, by x.

Now (if it please you) I wyll returne to my example of cattel. But I remembre youe sayde, I mighte not adde summes of sundrye thinges togither, and that might I see by reason.

M. Trouth it is, if you seke ẏ due sũ of any thinges. but if you only seke a bare sum, and haue no respect to ẏ thing, thã were it better to name ẏ sum onely w̃out any thing, as in saing 8 4 8, w̃out naming sheepe or ani thing else And like waies 1 8 6, naminge nothing. Nowe lete me see: howe canne youe adde those two summes

S. I muste firste set them so, that the two fyrste figures stande one ouer an other, and the other eche one ouer his fellowe of the same place: than shall I drowe a lyne vnder them both, And so like ways of other figures,

setting

settinge alwaies the greateste num- | 8 4 8
bre highest, thus. | 1 8 6

Than must I adde 6 to 8, whiche maketh 1 4. that is a mixte numbre; therfore must I take the dyget which is 4, and write it vnder 6 and 8. keping the article 1 in my mynde, thus.

8 4 8
1 8 6
4

Next that doe I come to the second figures, addynge them togyther, saying: 8 and 4, make 1 2, to which I put the one reserued in my mynde, and that maketh 1 3, of whiche numbre I write the diget 3 vnder 8 & 4, and keepe the article in my mynde, thus.

8 4 8
1 8 6
3 4

Than come I to the thirde figures saying: 1 and 8 make 9, and 1 in my minde maketh 1 0. Syr shall I write the cyphar vnder 1 and 8?

Maister. Yea.

Sch. Than of 1 0 I write the ciphar vnder 1 and 8, and kepe the article in my minde.

Ma. What needeth that, seeinge there folowe with no more figures?

S. Syr, I had forgotten: but I wyll remembre better herafter. Than seeynge I am come to the laste figures, I must write the cypher vnder them, and the article in a farther place after the cyphar, thus.

8 4 8
1 8 6
1 0 3 4

M. So now ye se that of 8 4 8 and 1 8 6 added together, there amounte 1 0 3 4

1034.

S. Nowe I think I am perfit in Addition.

M. That wyll I proue by this example.

There are twoo armies of souldiours: in the one are 106800, and in the other 94000: Howe many are there in bothe armies saye you?

S. First I set them one ouer on other, begynning with the firste nūbres at the right hand, thus. But the nether nūbre will not matche the ouer numbre.

$$\begin{array}{r}106800\\ 94000\\ \hline\end{array}$$

M. That forceth not.

S. Than doe I adde 0 to 0, and there amounteth 0, that must I write vnder the fyrste place, thus,

$$\begin{array}{r}106800\\ 94000\\ \hline 0\end{array}$$

M. Well said.

Sco. Then likewayes in the seconde place I adde 0 to, & there aryseth 0, whiche I write vnder the seconde place thus.

$$\begin{array}{r}106800\\ 94000\\ \hline 00\end{array}$$

Than I come too the thirde place, saing: 4 and 8 make 12, of which I write the digite 2, and keepe the article 1 in my minde, thus.

$$\begin{array}{r}106800\\ 94000\\ \hline 200\end{array}$$

Then adde I 9 to 6, whyche maketh 15, to that I adde the article 1 that was in my mynde, and it is 16, I wryte 6, vnder 6, and 9, and kepe one in my mynd.

thus.

1	0	6	8	0	0
		9	4	0	0
		6	2	0	0

Ma. Why do you not write bothe figures, seeyng you are come to the laste couple of numbres?

Sco. Nay, reason sheweth me that I muste adde that article that is in my mynde, vnto the next figure of the ouer summe, thoughe there be no more in the nether summe.

Ma. That is well consydered: then do so.

S. Then saye I, 0 in the ouer summe, and 1 in my mynd make. 1, that I wryte vnder 0: Then foloweth there two in the ouer summe whiche hath none to be added to it, for there is none in the nether summe, nor yet in my mynde, therfore I thynke I muste wryte that euen as it is.

M. Yea.

Sch. Then doth my whole summe appeare thus.

1	0	6	8	0	0
		9	4	0	0
1	1	6	2	0	0

Maister. If you marke this, you haue learned perfitly the common addition of all summes which are of one denomination, so that ye obserue this also, that in Addition you muste haue two numbres at the leaste, or else howe can you say that you adde? And euer let the greateste numbre be written highest, for that is the beste way though it be not necessary.

And forget not this, that if you haue many numbres to adde together, you shall haue

oft

oftentimes an Article of a greater valewe then 1 0: somtimes 2 0, somtimes 3 0, sõtimes 5 more, yea paraducture 1 0 0. Therfore as you did with the article 1 0, so dooe with them, reseruing them in youre minde, and adding to the numbre nexte folowinge so manye as theire valower or valewe certaine is: that is to say, 2 for 2 0, 3 for 3 0. and so forth of other. But if the article bee 1 0 0, then muste you not adde the article to the nexte figures folowing, but to the thyrde figures from them, as I will shewe you anone by example. And if it chaunce the numbre to bee suche that it doe comprehende two sundrye articles, (that is one of tennes, and another of hundredes) then muste you reserue them bothe in youre minde, and adde the article of tennes to the figures that folowe nexte, and the artikle of hundredes, to the figure of the thirde place from thence.

Nowe take this example for I wolde adde these xij. summes in one, whiche I sett after this mãner. Then doe I begin to gather ỹ sũme of ỹ first figures whiche commeth to 1 0 7. for first I take 9 therx. times, and that is 9 0: then 9 and 8 is 17, ỹ is in al 1 0 7. of which

4889
4599
2299
3699
2399
4090
1099
3198
299
699
899
499
389

ſum I write the 7 vnder the firſt figures, and than haue I an article of an hundred in my mind, which other I muſt keepe in my minde till I come to the thirde figures, whiche are in the roumes of hundreds, or els I maye for feare of forgettinge write one vnder the thirde rowe of figures, makinge two lines, as you ſee here done. And then muſt I write the digets vnder the loweſt line: and this is the ſureſt way, when the ſūme is ſo great, that the additiō of one rowe paſſeth 1 0 0.

4	8	8	9
4	5	9	9
2	2	9	9
3	6	9	9
2	3	9	9
4	0	9	0
1	0	9	9
3	1	9	8
	2	9	9
	6	9	
	8	9	
	4	9	
	3	8	9
	1	1	
		1	
		5	7

Whan I haue ſo done, I muſte than come to the ſeconde rowe of figures, and adde them togither, which doth make 1 1 5, of which ſumme I write the dygite 5 vnder the ſame ſeconde rowe, and then I haue an article remaininge of two figures, of whiche the one (that ſtādeth for 1 0) muſte bee added to the ſeconde, or next place after them that I did laſte adde. And the other (that ſtādeth for 1 0 0) muſt be added to the thirde place from thence.

Sco. That is to ſay, the fourthe place from the firſte lyne or rowe of fygures.

Ma. Euen ſo. And then will the ſūme appeare thus. Then adde the thirde rowe of figures, with the two vnitees betweene the

lynes, and the summe amounteth to 5 0: of whiche I write the Cypher vnder the same third row, & the 5 vnder the next figures toward the left hande.

Then I adde the figures of the fourth rowe, with yͤ 1 and 5 that are vnder them, betwene the two lynes, and they make 2 9: then write I the 9 (that is the diget) vnder the fourth place, and the 2 that is the article, beionde it, toward the lefte hande. So those summes do make 2 9 0 5 7.

	4	8	8	9
	4	5	9	9
	2	2	9	9
	3	6	9	9
	2	3	9	9
	4	0	9	0
	1	0	9	9
	3	1	9	8
		2	9	9
		6	9	9
		8	9	9
		4	9	9
		3	8	9
	1	1		
	5	1		
2	9	0	5	7

Sco. This semeth somewhat harde, by the reason of so manye numbres togyther. Howbeit I thinke if I doe often proue euen with this same example, I shall be able to do so shortly with any other summe.

M. So shall you. For it is often practise that maketh a man quicke and rype in all thinges: But bicause of suche greate summes in whiche there may chaunce to bee some errour, I wyll teache you how you shall proue whether you haue done well or no.

Scholar. That wer a great help and ease.

M. Begin first with the highest numbre, and than all the other orderlye, and adde them togither, not hauing regard to theire places, but as though they were all vnities: and still

as your nūbre encreaseth aboue 9, cast away 9. Then go forth euer casting away 9, as often as it amounteth therto: And so doe till you haue gone ouer all the numbres that you entended first to adde, and whatsoeuer remayneth after suche addition & castinge awaye of 9, write it in some voide place by the ende of a line for the better remembrance, & then put togither ye figures that resulte of thaddition, styll castinge away 9 also. And thē that, that remaineth, write at the other ende of the line: & if those two figures be like, than haue you well done by lykelyhode: but if they be vnlike, then haue you missed. As for example in this presente summe: The fyrste figure of the ouer lyne is 9, let him goe: then 8 and 8 is 16, take awaye 9, & there remaineth 7, adde to it 4 that foloweth, and that maketh 11, from which if you take 9, there resteth 2. Then come to the nexte rowe, whose first & seconde numbre are 9, therfore ouerpasse them bothe, and take the 5 to the 2 which dyd remaine in the first rowe, that maketh 7, put therto the 4 folowing, ye maketh 11 thence take 9, and there remaineth 2. Nexte that go to the third line, whose two first nūbres you maye let passe, bycause they are nynes: then take the two 2, which with the other 2 that remained in the seconde rowe make 6. Then goe to ye fourthe rowe, who two first numbres let go, and take the 6 to th

6 th

6 that remained, & that maketh 12: take away 9, & there resteth 3, which with the 3 that is nexte, maketh 6. And so go throughe all the other numbres, and you shall finde ꝧ there remaynethe 5, after you haue caste away 9, as often as you finde it: therefore wryte 5 at one ende of a line in a voyde place, thus. 5 ———

Then take all the figures of the total summe which is vnder the loweste lyne, and caste away 9 as often as you finde it, as thus. 7 and 5 make 12, take away nyne, and ther resteth 3: to ꝧ yf you adde the 2 that is laste (for you may let go the 9) then dothe it make 5, which you muste write at the other ende of the lyne that you made in the voide place, and it will be thus. 5 ——— 5.

And than you see those two figures be lyke, whereby you may knowe that you haue done well and so may you proue in any other.

Scho. If it please you I will proue in an other summe.

Maister. with a good will

Scholar. Then will I take one of your former examples, which was this.

Fyrst in the highest lyne 8 and 6 mak 14, than 9 taken awaye, there remaine 5, to whiche I adde the 1 that folowethe, and that maketh 6. Then come to the seconde line, where

1	0	6	8	0	0
		9	4	0	0
1	1	6	2	0	0

I finde firste 4, which withe 6, maketh 10, from that I take 9, & there resteth 1, the nexte figure is 9, & therfore I let him alone: so finde I one remaynynge, which I set at the ende of a line thus, 1 ———

Than I come to the totall summe, & there I finde that all the figures put together, make 10, from which I take 9, and ther resteth 1 also, which I put at the other ende of the line thus, 1 ——— 1

And bycause they be lyke, I knowe that I haue well added.

Mast. So you knowe now bothe howe to adde two summes or more togither: and also how to proue whether you haue done wel or no: which thinge also you may doe by Subtraction. But bycause you can not yet skylle of it, I will let that passe tyll anone, & wyll teache you nowe howe to adde summes of dyuers denominations: which thinge can neuer be, but when ẏ one denomination is such that it containeth the other certaine tymes.

Addition of diuers denominations.

And yet you shall adde them to the other, not after this sorte as you dyd them that were of one denomination, but after suche a sorte as I wyll nowe shewe you, that is to saye.

Yf you haue a summe of dyuers denominations, then looke that ye set euery denomination by him selfe, with some note or fygure of hys denomination, as they be wonte to be writte

written: Then write youre other summes so vnder that firste, that euery one be set vnder the other of the same denominations. As for example: yf youre denominations be poũdes, shillinges, and penns, write poundes vnder poundes, shillinges vnder shillinges, & penns vnder penys, & not shillinges vnder penyes, nor pens vnder poundes.

Scho. Nowe that you haue spoken it, mee thinketh it needeth not to warne me of it, for it wer againste reason so to confound sũmes: but yet if you hadde not spoken of it, peraduenture I shoulde haue been deceiued in it.

Ma. If you do say it is so plaine, I wyll speake no more of it, but with an exaumple make the matter to appeare euidentlye. First one man oweth me 2 2 li. 6 s. 8 d. An other oweth me 5 li. 1 6 s. 6 d. And another oweth me 4 . li. 3 s. I wolde know what this is al together. Therfore muste I firste sette downe my greatest summe, & then the other, euery one vnder his denominatiõ agreeinge to the greatest summe, as here you see.

li.	s.	d.
22	6	8
5	16	6
4	3	

Then muste I begyn at the smallest numbres, (which must alwaies be set nexte the ryghte hand) and adde them together. And if the summe of them will make one of the nexte denomination, then muste I keepe it in my mynde tyll I come to that place, or els

 for

for more easinesse, write it vnder that place betweene the double lyne, & vnder that firste place muste I note the residue, yf there remayne any of the same denomination. But yf there remayne none, then neede I to write vnder it nothinge. And this is all that you muste marke in thys Addition: for all other thynges are lyke to the other manner of addition before mentioned. Therfore the chiefest poynte of this addition is, to know the valewes of common coynes & rated summes, As how manye shillinges be in a pound: how manye penies in a shillinge. Of whiche and of other lyke thinges, I will instructe you hereafter, in teachinge of Reduction. But nowe I maye not dysturbe your witte from the thing that we are about.

Therfore lette vs return to that former exãple, which I proposed of thre detters, whiche summes when I had set orderly, they stod thus with a double lyne vnder them.

li	s	d
2 2	6	8
5	1 6	6
4	3	

Then to adde them vnto one summe, I must beginne at the ryghte hande where the smaleste denomination is, and adde them togither firste, saying: 6 and 8 make 1 4. Now seenge these 1 4 are pennies, and that 1 2 pens make one shillinge, which is the next ualewer, I take away 1 2 from 1 4, & there resteth 2, which I write vnder, the penye

an

and for the other 12, which make 1 shilling, I write 1 under the title of shyllyngs, thus.

li	s	d
22	6	8
5	16	6
4	3	
	1	
		2

Than do I adde all the shyllinges togyther, & find them 25, to whiche I ad 1 betweene ye two lines, that maketh 26. but bycause that 20 shillinges do make 1 pound, I take away 20 from 26, and for that 20 I write 1 under the poundes betwene the twoo lynes, and ye other 6 that remaineth, I write under the shillinges, thus.

li.	s.	d
22	16	8
5	6	6
4	3	
1	1	
	6	2

Then come I to the poundes addinge theim all together, & find them to be 31: thereto I ad ye 1 betwen ye ij lines, & yt maketh 32. whiche sum I write down whole, bicause there resteth no greater denomination. and then my whole summe appeareth thus.

li	s	d
22	6	8
5	16	6
4	3	
1	1	
32	6	2

So ys my total sum 32 li. 6 s. 2 d. And this may you proue in an other lyke sum.

S. Then wyl I caste the whole charge of one monethes commons at Oxford with batteling also.

Master. So to, let me see how you can doe.

S. One

S. One wekes cōmons was 1 1 d. ob. q̄, and my battelynge that weeke was 2 d. q̄. q. The seconde weekes cōmons was 1 2 d. and my batlyng 3 d. The thirde wekes cōmōs 1 0 d. ob. & my batlyng 2 d. q. c. The fourth wekes cōmōs 1 1 d, q̄, & my batling 1 d, ob. c. These 8 sūmes wolde I adde into one whole summe, and ther fore I will set them one ouer an other, thus.

d.				
1 1	ob.	q̄		
2		q̄.	q	
1 2				
3				
1 0	ob.			
2			q.	c
1 1		q̄		
1	ob.			c.

But I hadde forgotten, I should haue set the greatest sum highest.

M. So is it cōmonly best. howe bee it here it forceth not: and in suche sūmes as this is, that goe by ordre of wekes, daies, or yeares, yt is better to keepe that ordre, then to alter them, and to sette the greatest nūbre highest, for that serueth for suche sūmes as go not by ordre.

Sc. Then if I haue set theim wel ynough, I will begin to adde them thus.

First of the smalest valewers at the right hand, which are called cees, I finde 2, and setting that 2 cees, doe make one q. I will write nothing vnder

d.				
1 1	ob.	q̄		
2		q̄.	q	
1 2				
3				
1 0	ob.			
2			q.	c.
1 1		q̄		
1	ob.			c.
			q.	

th

the cees, but wil write 1 q for 2 cees, vnder the kewes betwene the line, after this manner.

Than come I to the nexte valowers, where I fynde 2 q, and to them I adde the q, that is betwene the lynes, and so are they 3 q, but bycause 2 q, maketh one q̄, I wryte one q̄ vnder the farthings betwene the lines, & the q that remaineth, must I write benethe the nethermost line, vnder the kewes, thus.

Then come I to the farthynges, where I fynde 3, and the other q̄ that is beeweene the lines, maketh 4 farthinges: And bicanse 4 q̄ make iuste 1 peny, I shall write nothinge vnder ẏ farthinges, but muste write 1 vnder the pence, betweene the lynes.

	d.				
1	1	ob	q̄		
	2		q̄.q.		
1	2				
	3				
1	0	ob.			
	2			q.	c.
1	1		q̄.		
	1	ob			c.
			q̄.q.		
1			q.		

Next that must I adde the halfepence together, of whiche there are 3, but seinge that 2 ob. make 1 d, I must write 1 vnder ẏ pēs, betweene the lines: but how shall I do it, for there is 1 already?

M. Haue you forgotten how I did in addition of the greate summe before? you muste set it vnder the other, so shall they both stāde in 2, for yf you shoulde set it before or behinde

the

the other, they shoulde make 1 1.

Scho. I remembre it nowe, & I perceaue the reason. Then I wyll write 1 ob. vnder the halfepence, and for the other two halfepence, whiche make 1 d, I wryte 1 vnder the pens: Then come I to the pens, & fynde that there are of them 5 2, than put I to the the 2 betwene the lines, and y^t maketh 5 4 which amounteth to 4 li, 6 d: the 6 d, I must write vnder the pens, & the 4 li, I muste sette (I suppose) farther towarde the lefte hande by them selfe.

Master. Euen so.

Scholar. Than appeareth al my additiō, thus. And the summe is 4 li 6 d ob. q.

d				
1	1	ob.	q̄	
	2		q̄.	q
1	2			
	3			
1	0	ob.		
	2			q. c
1	1		q̄.	
	1	ob.		c.
	1		q̄.	q.
	1			
4 li	6	ob.		q

M. Now haue you don this wel. But tel me, why dyd you write kewe, ccc, thus: q, c, & not rather thus q^c, as the fashion is?

Scholer. Because I thoughte it was the best way for due gatheringe of euery denomination by him selfe.

M. So was it in deede. Well now, can you tell howe to proue this addition, and such other like of diuers denominations: and trie whether you haue done well or no?

Scholer. I woulde I coulde.

Ma. That shall you do by thys meanes: fyrste as you did begine to adde, so recken a- gayne euery denominatiō by it selfe, & when ou fynde so many smale that do make anye ther denomination, let them go, and keepe n minde onely the residue that will make no reater denomination, & loke whether there e any suche lyke valewe vnder the neythere yne, and if there be, you haue wel done. and o go forthe frome one denomination to an o- her, vnto the ende.

But here must you note, that in gatherynge f the summes ye muste recken those figures hat are written betweene the lynes wyth hem that are written aboue them. as for an ample. I wil examine that sum that I did st adde, which stode thus.

l	s	d
2 2	6	8
5 1	6	6
4	3	
1	3	
3 2	6	2

yrst I finde 6 and 8 why e maketh 14. from why- e I take 12, bicause it ma eth one of the nexte deno- ination, and there remai- eth 2, and vnder that place I se a lyke fi- ure, therfore I know that wel to be done. hen come I to the s. where I fynde 1, 3, 6, & 6, that maketh 26, I cast awaye 20 they make an other denomination, that to say poundes: and the 6 whiche remay- eth, is lyke to the 6 that is written vnder em beneth the lowest lyne, therfore that well done also. And thence I go to poun-

des

des where I fynde 1,4,5 2 2, ꝧ is 3 2, to whyche summe agreeth an other lyke vnder it. Therefor I iudge all well done

Schol. I perceaue reason in this probati on. Nowe wyll I attempte the same in the summe that I dyd adde, which whan I hadde ended addyng, stoode thus.

Fyrst amongest the cees I fynde but twoo, whiche make 1 q, euen, therefore there muste nothynge bee vnder the line for theim: and amongest the kewes I fynde 3, of whiche two make 1 q̄, therfore I let theim goo, and the one q, that is lefte, hathe an other lyke vnder his place, therfore that is wel done.

	d			
1	1	ob.	q̄.	
	2		q̄.	q.
1	2			
	3			
1	0	ob.		
	2			ob. q.
1	1		q̄.	
	1	ob.		
	1		q̄.	q.
	1			
4ß	6.	ob.		q.

Than the farthynges are iuste 4, whiche make 1 d. and therfore I let them go. Amongest the halfe pence there is one odde (for 2 must I caste away, because they made one penny) vnto it aunswereth a lyke summe vnder it. The pence are 5 4, from whiche I take away 4 8, that make 4ß. ⁊ the 6 remaining agree to a lyke fygure set vnder them. and laste of all remayneth the 4ß, whiche the abiect pence dyd make: so I perceaue that I haue wel doone. Nowe this will I not forget

But will thys examination serue in all Addition?

Ma. It serueth for all Addition of sundry denominations, yf the addition be made with two lynes, (as these were) els it will not serue, bicause that those summes which are here added betwene the lynes in Addition by one lyne, are vnderstanded and not writtē, but I let that wayes passe, bycause as it is common, so is it more deceiuable then thys waies, namelye if a mans memorie be either dulle, other troubled.

Scholar. Yet it were good to knowe that waies also.

Mast. If you desire to know it, this it is in fewe wordes: Do euery thing as you dydde in this sorte of Addition, saue ẏ where you made here two lines, you shall make there but one: and those summes that you did here write betwene the lynes, you must keepe in your memorye, and them (as you dide here) adde one when you come to his place. An other forme of Addition.

Scho. Than they differ not but in this, ẏ this Addition wᵗ two lynes leueth nothinge to memory, but writeth doune all: and the other waye committeth certaine numbers to memory, as you taught me in the first examples of Additon of smale summes of one denomination. But what yf a man vse it (as you say men doe commonly) how shall it be examined?

Maister. Seing you are so desirous of it, I will shewe both an example of the addition, and also the manner to examine it. I propose these iij. summes to be added, and I gather first the pence, as I did in the other sort, and I finde of them 8, 3, 9, ye is 20 of whiche summe I bate away 12, which make 1 s. & kepe ye 1 in my mind, and the rest, ye is 8, I write vnder ye pēs Then do I adde the shillinges together, and finde of thē 6, 7, 8, that is 21, wherof I bate 20, that make 1 li. which I kepe in mind; and to the other 1 that remaineth, I adde that one that came of the pens, and was in my minde, which make 2, and them I write vnder the shillinges.

li	s	d
10	8	9
6	7	3
4	6	8

li	s	d
12	8	9
6	7	3
3	6	8
22	1	8

Then doo I recken the poundes together 3, 6, 12, that is 21, and to thē I adde the 1 in my mind that remained of the shillinges, which make 22, them doe I write vnder the poundes, and then my summe totall appeareth to be 22. li. 2 s, 8 d.

Another forme of proofe.

Nowe to examine this summe, and all such lyke, you shall doo thus. Firste begin at the lefte hande with the poundes, and take from them that are aboue the lyne, 9, as of-

ten

ten as you can: then that that remaineth, shall you double, and ioyne it with the shillinges, and take away 9 from þ as often as you can, and whatsoeuer remaineth ye shall take for it three times so much, and put to the pence. than take from all that sum 9, as often as you cā, & what so remaineth after you haue withdrawen 9 as often as you can, write þ at thende of a line, as I taught you in þ other addition.

And then come to the summe vnder the line, beginninge with the poundes, and doe euen as you did with the summes aboue the line, till you come to youre pennies, and if the figure of the summe that remaineth after castinge away 9 (as often as you can) do agree with the other that remained before of the other summe, whiche you dyd write at the ende of the lyne, than haue you done well, else not. and for an example I will examine that laste summe whiche was thus set.

Firste I shall begin at the lefte hande with the pounds putting them together, which make 21, in whych summe I fynde 9 twyse, (for twise 9 is 18) that I deduct, and there remayneth 3: that 3 muste I double (as I sayd) bicause it is the remainer of the poundes, and it will be 6. Than gather I

l	s	d
12	8	9
6	7	3
3	6	8
22	2	8

the somme of the shillinges which is 2 1, to the whiche I adde the aforesayde 6, and than is it 2 7, wherein I fynde 9 three times, and there remaineth nothing. Thys remayner shoulde I take three times: but three tymes nothynge, is nothinge: therfore in this place is there nothinge lefte to be added to the pennies. Wherfore I must take ẏ summe of pennies alone, which is 2 0, frō thence if I take 9 twise, there remaineth but 2, which I put at the ende of a line, thus, 2 ———

Then I come to the poundes of the vnder nūmbre or totall summe, and there fynde 2 2, from which I take away 9 twyse, and there remaineth 4: that 4 I double, and it is 8: then do I adde that 8 to the shillinges, and it maketh 1 0. from which I withdraw 9, and there resteth one: then do I take that 1 three tymes, & it maketh 3, which I adde to the 8 d. and maketh 1 1. from whiche if I bate 9, there resteth 2, which is equal to the nūmbre noted at the ende of the lyne: and therby I perceiue that I haue done well.

Scho. But I do not se the reason of thys.

Maist. No, no more do you of manye thynges els, but hereafter wyll I shewe you the reasons of all Arithmeticall operations. for this I iudge to be the best trade of teachinge, firste by some briefe preceptes to instructe a learner somewhat in the vse of the arte, before he learne the reasons of the arte, and

then

then may you afterwarde more soner make hym to perceiue the reasons : for harde it is for to occupie a yonge learned witte with both the arte and the reasons of it al at ones: howbeit hee shall neuer bee cunninge in dede in an arte , that knoweth not the reason of euery thing touching it. But for this worke, bicause the reason is easy, I will shewe it you nowe . You knowe that if one pounde do re maine, it beinge tourned into shillinges, wold make 2 0 s. in which numbre there is 9 conteyned twise, and 2 s. beside . And therefore for one pounde you shall take 2 s, and so for euery one pounde 2 s.

Scho. I see it well, for yt there remayned 7 li, after the nynes were cast away, I must take 1 4 s. for that 7 li. And so haue I caste awaye 1 4 tymes 9 s. and yet remayneth of euery pounde 2 s, which maketh 1 4 s.

Maister. Like waies in shillinges, whiche containe 1 2 d. for euery shilling, if you abate 9 pence, there resteth 5 pence.

Sc. It is playne ynough . And so if 5 shillinges do remaine, I must take for it 1 5 d: that is 3 pence for euery shillinge, & yet in y so doinge, I haue cast away 5 times 9 pence.

Ma. Other workes haue as good reason, but I will not stande aboute yeldinge reasons nowe.

Sc. Yet one thing more I pray you shewe me, why did you write your numbre that re-

 mained

mained (after you hadde withdrawen all the nynes) at the ende of a line? for I sawe no reason why that line did serue.

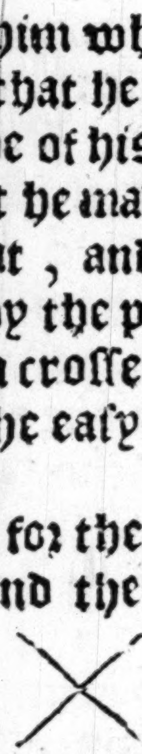
M. Did you euer marke a carpenter whan he wrought?

Scholar. Yea many times.

M. And haue you not seene him whan he hath taken measure of a boorde, that he hathe pricked it, and hath with a twitche of his hand drawen a line from the prick that he made?

Scho. Yes, I haue marked that, and haue seene some make 3 or 4 lynes by the pricke, and som also haue I seene make a crosse by it, but that I perceiued, was for the easye fynding of their pricke.

Ma. And euen so is this lyne for the easye findinge of youre remainer, and therefore some we make a crosse thus.

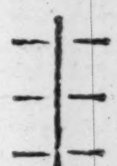


And set the one remainer aboue the crosse, and the other vnder ẏ nether parte of the crosse, as if I shoulde set my two remayners thus.

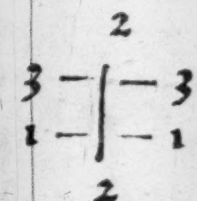
But there is an other sorte of profe of Addition, to whiche the crosse serueth more meter: And that is when the addition is of diuers denominations, and I woulde examine euerye denomination by it selfe, whiche waies though it be not much vnlike to the firste proofe that I broughte of such diuers summes, yet I wil declare it, least you should think that I wold hide it frō you.

An other sorte of proofe.

Yo

You muste make so manye lynes in youre crosse as you haue sundrie denominations: as if you haue but two denominations, then you may make it thus, that the ouer part and the nether parte may serue for one denomination, and the twoo sides for the other.  And if you haue thre denominations, as poũds, shillings, & pennies, then must you make thre lynes thus.  The vpright line maye serue for the poundes, and the highest thwart line for shillinges, & the lowest for pẽnes. as for example I will take a sum thus added,

li	s	d
16	12	5
12	8	1
9	2	7
38	3	1



For the proofe of the whiche, bicause it conteyneth thre denominations, I muste make a crosse of thre lynes, thus. Then I recken firste at the right hande the pennies: 7, 1, 5, make 13, from which I take 12 for the next denomination, that is to say, a shillinge, and there resteth 1, whiche I muste write at one ende of the nether twhart line.

After that I gather the sum of the shillinges, 2, 8, 12, which make 22, to them I put one that I toke of the pennies, and that maketh 23, from those I take 20, the quãtitie of the next greater denomination, that

 is

is to saye, a pounde, & ther resteth 3, whiche I write at thend of the highest thwart lyne.

Thirdly I adde togither the poundes, 9, 12, 16, which make 37, to them I adde the 1 that came of the shillinges, and than there is 38, wherin I find 4 times 9, and 2 ouer, that 2 I write on the vpright lyne.

That done, I come to the totall summe, and examine it, beginning at the pennies, where I finde but 1, and can not take 9 frō him, therfore I set him at the other end of ỹ nether thwart line: than come I to the shillinges, where I finde only 3, whiche because it is lesse thā 9, I set it at the other ende of ỹ line of shillings, ỹ is, ỹ ouermost thwart line.

Laste of all, of the 38 li, I take oute 4 tymes 9, which is 36, & there remaineth 2 which I write vnder the vpright line.

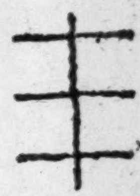
Then I considre euery nūbre, comparinge it to the numbre that is against it, & bicause I finde thē to be euery one like his matche, I knowe that I haue well done.

Scho. This crosse I perceiue doth serue for those thre denominations, pounds, shillinges, pennies. but what yf I had ob. q̄. q. and c?

Ma. Youe thinke you be at Oxforde styll, you bring forth so faste your q. and c. Theire lines, as I haue sayde, dothe serue for thre denominatiōs, suche as they be: as here they do serue for poūds, shillings & pennies: but if ye haue no poundes in your summe, then

then maye thei serue for shillings , pennies & halfe pennies, yea for q̃, q, & c, if you haue no greater denomination , so that you remembre that ẏ vpright line serueth for the greatest denomination , & the highest thwarte line for the next, & the lowest for ẏ leaste.

And so yf you haue fower denominations, you muste make your crosse wͭ so many lines. And if ẏ your sum be of more denominations, make so many lines in youre crosse. And thus wyll I make an ende of Addition.



Examples of Addition.

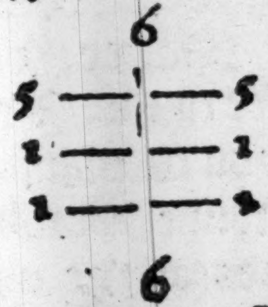
li	s
262587	6
41635	12
28124	2
471	4
332818	4

li	s	d
340	17	10
28	6	8
13	13	0
382	17	10

The proofes.

An other example.

li	s	d	q.
2		6	2
2	3	4	
	10	2	3
4	14	1	1



Scholer.

THen haue I learned the two fyrſte kyndes of Arithmetike, nowe (as I remembre) dooeth folowe Subtraction, whoſe name me thinketh doeth ſound contrarie to Addition.

Subtraction.

Maſter. So is yt in dede: for as Addition encreaſeth one groſſe ſumme by bringing many into one, ſo contrarie waies Subtraction diminiſheth a groſſe ſumme by withdrawing of other from it, ſo that Subtraction or Rebating is nothing els, but an arte to withdraw and abate one ſum from an other, that the Remainer may appeare.

Scholer. What call you the Remainer?

Maſter. You may perceiue by the name.

Sc. So me thinketh: but yet it is good to aſke the troth of al ſuch thinges, leaſt in truſting to mine owne coniecture, I be deceiued.

Maſter. So is it the ſureſt way. And as I ſee cauſe, I will ſtill declare thinges vnto you ſo plainely, that you ſhall not neede to doubt. Howbeit, if I do ouer paſſe it ſometimes (as the manner of men is to forget the ſmale knowledge of theym to whome they

ſpeake

speake) than do you put me in remembraunce your selfe, and that way is sureste.

And as for this worde that you laste asked me, take you this description. The Remainer is a summe lefte after due working, whiche declareth the excesse or difference of the two other numbres, as if I woulde deducte 14 out of 18, there should remaine 4, whiche is called the remainer, and is the difference betweene those two numbres 14 & 18. Remainer.

Sc. I perceiue than what Subtraction is, now resteth to know ye arte to worke by it.

Maister. That shall you doe by this meanes: Fyrste you muste consider, that if you shoulde go about to rebate, you muste haue two sundrye summes proposed, the fyrste, whiche is your grosse summe or summe totall, and yt muste be set highest, and then the rebatement or summe to be withdrawen, whiche muste be sette vnder the fyrste (whether it be in one parcell or in many) and that so, that the firste fygures be one iust ouer an other, and so the second and thyrde, and all other folowinge, as you dyd in Addition: then shall you drawe vnder theym a line, and so are youre summes dewly set to begyn your working.

Then begyn you at the ryghte hande (as you did in Addition) and withdrawe the nether numbre out of the higher: and if there remayne any thinge, wryte that right vnder them beneth the line: and yf there remayne

nothinge

nothing,(by reasõ that the two figures were equall) then wryte vnder them a cyphre of nought. And so doe you with all the other fygures, euermore abatynge the nether oute of the higher, and write vnder them the remainer styll, tyll you come to the ende. And so wyll there appeare vnder the lyne what remayneth of youre grosse summe, after you haue deducted the other summe from yt, as in this example.

I receiued of your father 48 li, of whych I haue laide oute for you 36 li, nowe would I knowe what doth remaine? and therfore I set my numbres thus in ordre: Fyrste I wryte the greatest summe, and vnder him the lesser, so that the fygures at the ryght side be euen one vnder an other, & so that other thus.

li	
4	8
3	6

Than doe I rebate 6 out of 8, & ther resteth 2, which I write vnder the right beneth ÿ line, thus,

li	
4	8
3	6
	2

Than I gooe to the seconde fygures, and do rebate 3 out of 4, where there remaineth 1, which I write vnder them right, & then the hole summe and operation appeareth thus.

li	
4	8
3	6
1	2

Wherby yt appeareth, that y I withdraw 36 out of 48, ther remayneth 12.

Scho. Nowe will I proue in greater summe: And I will Subtract

2367924

2 3 6 7 9 2 4 out of 3 4 6 8 9 4 6. Those summes I set in ordre, thus.

3 4 6 8 9 4 6
2 3 6 7 9 2 4

Then do I beegin at the ryghte syde, and deducte 4 out of 6, and there resteth 2, whiche I write vnder them. Then goe I to the seconde figures, and withdrawe 2 out of 4, and there remaine 2, which I set vnder theym also: than I take 9 out of 9, and there resteth 0, whiche I write vnder theim: for you saye, that yf the fygures bee equal, so that nothinge remayne, I muste write this cyphre 0 vnder theim.

Ma. It was wel remembred: nowe go forth.

Schol. Then come I to the fourth place, and drawe 7 out of 8, and there remayneth 1, which I write vnder theym also. Then in the fift place I take 6 from 6, and there remayneth nought, for it I write vnder them a cyphre 0: Than in the vj place 3 rebated from 4, there remayneth 1, whiche I write vnder theim: And likewise in the vij. and laste place, 2 taken from 3, ther is left 1, which I write vnder them. so haue I done my whole workinge, and my summes appeare thus.

3 4 6 8 9 4 6
2 3 6 7 9 2 4
1 1 0 1 0 2 2

Wherby I see, that yf I rebate 2 3 6 7 9 2 4 oute of 3 4 6 8 9 4 6 there remaineth 1 1 0 1 0 2 2.

M. This is well done.

And that you maye be sure to perceaue fullye

the

the arte of Subtraction, let me see howe can you subtracte 5 2 9 8 4 7 3 2, out of 8 2 5 0 0 0 3 4 5 6.

Sc. Fyrste I set downe the greatest sum, and after that I write vnder it the lesser nūbre, beginning at the ryght syde: and then my figures wil stand thus.

Thā take I 2 frō 6, & the reste is 4,	8 2 5 0 0 0 3 4 5 6
	5 2 9 8 4 7 3 2

whiche I wryte vnder them, then do I withdrawe 3 from 5, and there remaine 2, which I write vnder them: Then take I 7 out of 4 but that I can not, what shall I nowe do?

Note. Mast. Marke well what I shall tell you now, how you shall do in this case and in all other lyke. Yf any figure of the nether sum be greater then the fygure of the sum that is ouer him, so that it can not be taken out of the figure ouer hym, then must you putte 1 0 to the ouer fygure, and then consydre howe muche it is, and out of that whole summe withdrawe the nether fygure, and write the rest vnder them. Can you remembre this?

S. Yes, that I truste I shall. Nowe then in mine example where I shoulde haue taken 7 out of 4 & coulde not, I put 1 0 to that 4 which maketh 1 4, frō it I take away 7, & ther resteth 7 also, which I write vnder them.

Note. M. So haue you done wel. but now must you marke an other thinge also: that whensoeuer you do so put 1 0 to any figure of the

ou

ouer number, you must adde one styll to the figure or place that foloweth next in the nether lyne, as in this example there foloweth 4, to which you must put 1, & make him 5, and then go on as I haue taught you.

S. Then shall I saye: 4 and 1 (which I must put to him for the 10 that I added to 4 before) made 5, which I should take out of 3, but that can not be, therefore must I put to it also 10, and then it wil be 13, frõ which I take 5, & there resteth 8 to be written vnder them; and bicause of that 10 added to the 3, I must adde 1 to 8 that foloweth in the nether lyne, and that maketh 9 which I should take out of 0, and can not, therfore I put thereto 10, and that maketh 10: from 10 I take 9, and there remaineth 1, which I write vnder them.

Then doe I adde 1 likewise to the next figure beneth, which is 9, and that maketh 10: that 10 shuld I take out of the figure aboue, but I can not, for it is 0, therefore I put 10 to it, & so take I 10 out of 10 and there resteth 0 to be written vnder them: the come I to the next figure whiche is 2, and to him doo I adde 1, which maketh 3, that 3 I can not take out of nought, therfore of that nought I make 10, and thence doe I take 3, so remayneth there 7 to be written vnder them: Likewise do I put 1 to 5 that foloweth, and then is it 6, that woulde I take

take out of 5, and can not, therefore I take 1 0 to that 5, and make it 1 5, from which I rebate 6, and there remaineth 9, which I wryte vnder them: now haue I spent all the nether figures, & what shall I do more?

M. You shoulde haue added 1 to the nexte fygure folowinge, (yf there hadde been any) because you added 1 0 to the laste figure before of the ouer lyne: But seeing there is no fygure folowinge, you muste adde that 1 to the place folowinge, and then deducte that 1 from the numbre aboue.

Sc. Then shall I say, bycause I borowed 1 0 to the ouer 5, I muste put 1 to the next place beneth that is vnder 2: then muste I subtract that 1 from 2, and there resteth 1 to be written vnder that 2 in the nynth place. Nowe I haue no more to subtract, for there is neuer a fygure remaininge beneth, nother yet anye vnitee to bee added, bicause I borowed not 1 0 to the figure laste before, and yet is there 8 remaining in the ouer lyne, whiche (I thinke by reason) shoulde be set at the ende of the figures in the loweste rowe whyche is vnder the lyne, for bicause there was nothing taken from it.

Maister. That is well considred, and reason teacheth so in deed.

Scholer. But syr I beseech you, shall I alwayes when anye numbre so remaineth alone (as this 8 didde) write him vnder the lyne

straygh

straight against his owne place?

M. Yea what else? whether they bee one or manye: and this well remembred, you haue sufficiently learned Subtractiō. How be it, because of certaine things that might deceiue you, if you did not take good heede to your working, I will propose to you an other example of many numbres to be subtracted, as thus.

I receaued of a frende of myne to keepe 2869 crounes, of which at one time I deliuered him againe 500: at an other tyme 368: & an other time 440: and an other tyme 80 and an other tyme 64: now would I knowe how many dothe reste behynde. Therefore firste I set downe my grosse sum, and underneth it I set all the parcelles, thus: and vnder them a double line.

2869
500
368
440
80
64
1452
1417

Then firste I begynne at the firste place, and gather togyther the sum of all those lynes (saue the ouermoste) in theire firste figures, and so doe I wyth all the figures of the seconde place, and so forthe, as I dyd in Addition, saue that I leaue out the highest rowe of numbres, and that summe so gethered betwene the lynes, the I subtract out of the highest rowe of nūbres, and the remaincr do I set vnder the ne-

 thermost

thermost line. a for exãple.

I set the summes as before; than doe I gather the first figures together, where I fynde but 4 and 8, that make 12, for three ciphres encreaseth no summe in addition, as you learned before of ẏ 12, therfore do I write the digete 2, and keepe the article in minde, till I come to the seconde places, where I finde, 6, 8, 4, 6, that make 24, to theim I put the article in my mynd, & it is 25, of which I write 5 vnder the seconde place, & keepe the diget 2 in my minde for the third place, where I finde 4, 3, 5, that make 12, to the which I adde the 2 in my mynde, and that maketh 14, thereof I write the 4 vnder the thirde place: & bicause there remaineth no more figures to be added, I write the diget 1 in the fourth place, as you see in the example.

2	8	6	9
	5	0	0
	3	6	8
	4	4	0
		8	0
		6	4
1	4	5	2
1	4	1	7

Than come I to subtractinge of thys summe, and I shall subtracte the summe betwene the lines from the ouermoste summe, saying: 2 from 9, remayne 7, to bee wrytten vnder them beneth the lowest line: than in the seconde place I take 5 from 6, and there resteth 1, to be written vnder them. Than in ẏ thirde place, 4 from 8 resteth 4. Laste of all in ẏ fourth place, 1 from 2, remaineth 1

And

And thus I see that after those 5 summes are subtracted from 2869, the Remainer is 1417.

S. This I perceiue: but is there no shorter waie and more speedier?

Ma. Yes, when you are a while exercised in it: for you may as faste as you can gather the numbers togither, withdrawe them out of the highest summe, if so be it, that all the parcelles which you doe gather, doe not exceede nyne, but and yf they exceede nyne, then muste you subtracte onely the digette that is in it, and reserue the article tyll the next place, where you shall adde it with the other figures, and so subtracte the whole out of the figure aboue them: but and if in thys place the summe of the parcels do excede 9, then (as I sayde before) subtracte the diget onely, & reserue the article to the next place, and so still go forth, till you haue ended your working.

As for example: in the laste summes proposed, I gather firste in the firste place 4 and 8, that maketh 12, of which I deducte the digite 2 out of 9, and write vnder the Remainer, which is 7, and the article 1 I kepe in my mynde. Then in the second place I gather the parcels 6, 8, 4, 6, which amount to 24, to that I adde the article 1, which I haue in my mynde, and then is it 25: then do I take 5 (that is the digitte in

this numbre) from 6, that is in the seconde place of the highest summe, and there remayneth but 1 to bee written vnder theym, and now do I keepe the article 2 in my minde still. Than in the thirde place, 4 3 5, maketh 1 2. and the article 2 in mynde, maketh 1 4: then take I 4 (whiche is the digete) from 8 that is ouer them, and there resteth 4, whiche I write vnder theim. Then haue I the article 1 yet in my mynde, whiche I sholde adde to the parcels nexte folowing, but seeinge there is no numbre folowing, I take that diget alone, and deducte him out of the nexte summe aboue, which is 2, and than is ŷ remainer 1, which I write in ŷ fourth place vnder 2. Loe, now haue you a shorter way.

Sc. I like bothe wayes well, and I perceaue bothe well. yet as in the one, the working seemeth somwhat longe, so in the other it leaneth verye muche (mee semeth) to remembrance, and therfore may cause errour quicly, excepte a man haue a quicke and an exercised remembraunce.

M. What woulde you then haue suche a waye that shoulde not bee so long as the one, nor so shorte as the other?

S. Yea, if there were any suche.

M. Than do thus: stil as you gather your parcels when they excede a diget, and maketh him 1 0 or more, take the article, and wryte him betweene ij lynes (as in the first exāple) vnder

vnder the next place towarde the lefte hande, & then deduct yͤ digete from yͤ figure that is ouer him, & write yͤ remainer. And then whē you gather the next parcels, you shall adde to them the figure that is vnder them betweene the two lynes. And if it excede 9, doe as I saide before, write the article vnder the nexte place betweene the lynes, and subtracte the digit from yͤ figure that is ouer those parcels: and if that all the prcelles togither & the numbre betweene the lynes we make but a digete, then deducte it wholly from the figure aboue: as in this example.

I would subtract out of 40308964,

	40308964
these three parcells	20003428
	10001342
	10101461
	———————
Therfore I set them	———————

fyrste in ordre due: and then I gather the parcels of the fyrst place, which are 8, 2, 1, yͭ is 11: of which I take away the article, and set him vnder the seconde place betweene the lynes, and the digete 1 that remayneth, I deduct: out of 4, and there resteth 3 to be written vnder the firste place benethe the lowest line. Then come I to the seconde place, and gather the parcels of it, 6, 4, 2, and the one betweene the lynes, which make 13. of which I take the article, and set him vnder the thirde place betweene the lines, and the

digitte 3 I take from 6, and there remayneth 3, whiche I write vnder the seconde place benethe the lowest lyne. Then in the thirde place I finde 4, 3, 4, whiche with the 1 betwene the lynes, doe make 12, therfore I write the article againe vnder the fourth place, and the digete 2 I take from 9, and there remaineth 7, whiche I write vnder them benethe the lowest lyne.

And then come I to the fourthe place where I gather 1, 2, 3, and the 1 betwene the lines, that maketh 7, whiche because it is but a digit, I plucke from 8, and the Remainer is 1, and must be written vnder them in the fourth place. After that come I to the fifte place, where are onely three cyphars, which make nothinge, then shoulde I take that, that is to saye nothing, from the figure ouer them, which is also a cyphre, therfore I muste saye thus: Yf I take nought from naught, there remaineth nought: so muste I write a ciphre vnder theim. Then in the syxt place I finde but 1, which I take out of 3 ouer him, and the Remainer is 2, that must be writen benethe the lowest line in the sixte place. So goe I to the seuenth place, where I find only cyphers, and in the grosse summe ouer them a cyphre also, therefore must I write their remayner (whiche is nothinge) with a cyphre also. Then in the eighte and laste place, I gather 1, 1, 2

that make 4 , whiche if I take out of that 4 that is ouer theim, there will nothing remaine. And that must bee noted wich a cyphre beneth the loweste lyne, as I haue often said. and so haue I ended my worke , and the fygures stande thus.

Sch, Sir I remembre you taught me that cyphres shold not come in the last place, for bycause they serue only to encrease the valewe of other fygures whyche folowe theim , and serue not for those fygures that go before them; and nowe in youre example you haue set two cyphres in the two laste places.

4	0	3	0	8	9	6	4
2	0	0	0	3	4	2	8
1	0	0	0	2	3	4	2
1	0	1	0	1	4	6	1
				1			
0	0	2	0	1	7	3	3

Mast. I commende you for youre remembrance. And truth it is, I shoulde not haue set them here , but onlye bicause that I woulde make you playnlye to perceaue the arte of Subtraction. Therfore seeinge you are nowe perceiue it , whansoeuer you shoulde wryte downe a cyphre, looke whether anye other figures be yet behinde. And if not , than let go the cypre also , for it needeth not to wryte him in any latter places, where no other fygure doth folow, except it be (as I did) to teach the vse ob Subtraction the playner. Therfore my figures must stand thus. whan

I

I haue ended my work.

S. So I wold thinke by that you taught me before. And nowe I beleue I coulde subtracte any like summes

4	0	3	0	8	9	6	4.
2	0	0	0	3	4	2	8
1	0	0	0	2	3	4	2
1	0	1	0	1	4	6	1
					1	1	1
	.	2	0	1	7	3	3

Maist. So may you, yf you haue marked what I haue taught you. But because this thinge (as al other) must be learned surely by often practise, I will propounde here ij. exāples to you, wherin if you oftē exercise your selfe, you shalbe ripe and perfect to subtract any other sum lightly, for in thē is cōtained all ỹ obseruances of whole numbre. And because you shall perceiue somwhat bothe how to doe it, & also whether it be well done whan you haue proued to doe it: therefore haue I written vnder them both the remainers.

3	0	8	9	6	4		1	2	5	6	1	4
1	0	3	1	4	5					3	4	2
1	0	2	5	9	7					6	8	1
1	0	1	0	2	4					2	0	1
		1	1							1	1	
		2	1	9	8		1	2	4	3	9	0

Sch. Syr I thanke you. But I thynke I might the better doe it, yf you did shewe me the working of it.

Maist. Yea but you must proue your selfe to doe some thynges that you were neuer taught

taught, or els you shal not be able to do any more than you were taught. And that were rather to learne by rote (as they call it) than by reason. And againe, there is nothing in this example or any other of whole numbre, but I haue taught you the rules of them already.

Scholer. Then I truste by practise to attaine the vse of yt. And ys thys all that I shall learne of Subtraction?

M. Yea, sauing that (as you haue seen in Addition) there are numbres of diuers denominations, in whiche the working is not muche vnlike, yet without some instructiōs be geuen of it, yt mighte seeme to a learner more difficult, then in deede it is. Therfore I will brifly shewe youe the vse of it onely, by one example or two.

A certaine man owed to me 14 li, 12 s, 8 d of whiche he paied me at one tyme 4 li, 6 s, 8 d: at an other time 3 li, & at an other tyme 2 li, 3 s, 4 d, and laste of all, 6 s, 8 d: Nowe woulde I knowe, what remaineth vnpaied yet. therfore I set my summes thus.

Sc. Syr I praye you, why do you write 2 li? for the common speache vseth rather to say 40 s?

M. We must here vse the denomination that ys greatest in any sum, so that wee maye not

li	s	d
14	12	8
4	6	8
3		
2	3	4
	6	8

write according as we vse to speake, sayinge, 1 6 d. 1 8 d: or like waies, 7 grotes, 8 grotes 1 4 s, 4 0 s, 4 8 s, and such other, but wee must write euery denominatiõ ỷ is in any sum by it selfe, namely shillinges and poundes. So must we write for these summes now named, 1 s, 4 d: 1 s, 6 d: 2 s, 4 d: 2 s, 8 d: 1 li, 4 s, 2 li, 8 s: and so forth of other lyke.

Scholler. So that we may not write in Arithmetike pennies: when the summe amounteth to shillinges: nor shillinges whan the sum maketh poundes. Nowe if it please you, ende your example.

li	s	d
1 4	1 2	8
4	6	8
3		
2	3	8
	6	8
	1	
4	1 6	

M. Whã my summes are so set as I shewed, than muste I beginne with ỷ smaleste denominatiõ, saying: 8, 4, 8, are 2 0. whiche summe bycause yt is pence, and 1 2 pẽnies do make 1 s, I must take from that 2 0 (which cõmeth of the three parcels 1 2, and for them write 1 betwene the lines vnder the shillynges. than the 8 d. that remaineth, I take out of the highest summe, whiche is 8 also, and then remaineth noughte. wherfore vnder the pence I wryte nothing. Than come I to the shyllinges, and gather the parcels 6, 3, 6, which with ỷ 1 betwen the lynes, make 1 6, that must I take out of the

the summe that is ouer it: but seeinge that sum is but 1 2, I can not take 1 6 out of 1 2 I must borowe 1 of the 1 4 li. and put to the 1 2, & that maketh 3 2, for 1 l. is worth 3 0 s. thā take I 1 9 out of 3 2, and ther resteth 1 6 to be written under the shillinges. Then come I to the poundes, whose parcelles are 2, 3, 4, ỷ is in al 9, & 1 more must I adde therto, bicause of the 1 that I borowed before unto the 1 2 s. and then is there 1 0, which I must take out of 1 4, so doeth there remaine 4, to be written under the poundes: so doeth my remainer appeare to be 4 li. 1 6 s.

S. This doo I perceiue very well, and if there be none other thinge to be learned in subtraction, then may I come to Multiplication, for that you reckoned to bee in ordre nexte.

Ma. We haue done in dede with the art of Subtraction, as touching the working: but yet before we go to Multiplication, I wil instruct you how to examine your worke, whether it be wel done or no, and that is by casting away 9 as often as you can finde it, as you did in addition, sauing that you must here examine the highest number alone, and note the residue of it at a lines ende, as you did in Addition. A proofe of Subtraction.

And when you haue done with the highest number, then examine all the other together, casting thence 9 as often as you can,

and if the remayner bee lyke the other, then haue you doone well.

But yf you haue diuers denominations in your sum, yet for them all shall you make but one seuerall lyne as you dyd in Additiõ, remembryng to beginne the examination at the greatest denomination, and to double the remayner of poundes, and tryple the remayner of shilings, as you did also in Addition.

As for a profe, I will examine this worke wher in the highest line I finde of poũds 14, from thence I bate 9, & there resteth 5, whiche I doe double, bicause they are poundes, & than are thei 10, therto I adde the 12, and it maketh 22, from whiche I take 9 twyse, and there resteth 4, which bycause they are shillinges, I triple, and then are they 12, thereto I adde the 8, and than are they 20, thence take I twise 9, and yet resteth 2, which I write at the one ende of a lyne thus. 2 ———

l	s	d
13	12	8
4	6	8
3		
2	3	4
	6	8
4	16	

Then I examine all the other parcels and the remainer together, euerye denomination by it selfe. And fyrste of poundes I fynde 4, 3, 2, 4, ỹ is 13, from which I take 9, there resteth 4, that do I double, and it maketh 8, to it do I put the shillinges, 6, 3, 6, 16, that is 31 (for ỹ one betwene the lines

must

muste not bee reckned, nor none in that space) and that maketh in all 39. Where hence I take 9 foure times, and there remaineth 3, that to I take three times, and it is 9, wherfore I cast it away: than to I take the pennies 8, 4, 8, ye maketh 20, frō which I take 9 twise, and there resteth 2, which I write at the other end of the profe line. And bicause I see that those twos numbres are equall, I saye that I haue well wrought.

And yf you will, you may make for euery denomination a lyne, as you learned in Addition: but then muste you begin your examination at the smalest denomination, as you dyd in Addition: for their proofe is altogether lyke, sauing that in Addition you examined the nethermoste summe alone, and all the other togither: and in Subtraction ye muste examine the highest numbre alone, and all the other toegther. And if you marke yt well, it is euen all one, for that summe that in Addition is loweste, in Subtraction is higheste. And that summe is called the Grosse or Totall summe.

Grosse or total sūes

Therfore yf you marke what I sayde in Addition, you maye easylye perceaue what is to be done for the profe of Subtraction. And to the entente that you may perceaue yt the better, I wyll shewe you an other profe of subtraction, and that shall bee by addition, thus. Draw vnder the loweste numbre, why che

An other proofe of Subtraction.

the is your Remainer, a line: then adde that numbre, and all the other that you did subtract before, together, and write that that amounteth, vnder thr lowest line: & if that sum that commeth therof be equall to the highest of the subtraction, then was the subtraction well wroughte, or elles not: as for example, in the last summes, which stoode thus.

First I adde 8, 4, 8, that maketh 20, whereof I take 12 awaie, because they make one shilling, and write for them 1 vnder y shillings: & the 8 that is left, I write beneth the lowest line: then adde I the shillings 6, 3, 6, 1, 16, y make 32: from which I take 20, and for it I write 1 vnder the pounds, and the 12 that remaineth, I write vnder the shillinges. Then come I to the poundes adding them together, which are 4, 3, 2, 14, that maketh 14: than do I write 14 vnder the li, & so haue I ended the addition. And I se that y lowest line of number and the highest bee lyke, wherefore I knowe that I haue well doone, for my figures appeare thus.

li	s	d
14	12	8
4	6	8
3		
2	3	4
	6	8
1	1	
4	16	

li	s	d
14	12	8
4	6	8
3		
2	3	4
	6	8
1	1	
4	16	
14	12	8

And thus nowe haue

I

I taughte you the arte of Subtraction, and the meanes to proue whether it be well wrought or not.

Nowe and you remembre, I omitted in teachinge the proofe of Addition one waye, whiche I sayde was by Subtraction.

S. Truth it is, & than was it deferred, bicause I had not thā learned the feate of Subtraction, wherby I shold haue proued it: but now I thanke you, I haue well learned the arte of Subtraction, and the proues of it, boothe by 9, and by addition. And nowe I woulde be glad to knowe, howe I may prooue Addition by Subtraction.

M. Then marke you this, when you haue ended your addition, take the numbres, all that you did adde, to the higheste summe, and deduct or subtract them from the grosse sum that doth resulte, & yf the remayner bee lyke to the higheste numbre, than haue you doone well, els not.

The profe of Addition by Subtraction.

As for example. I take one of the sūmes that I dyd adde before, whych was this that foloweth here.

1 0 6 8 0 0	
9 4 0 0	
1 1 6 2 0 0	

Then doo I come to the middle nūbre, and subtract that from ẏ nether nūbre, begynninge at the righte hāde, & fyrst I say, 0 out of 0, there remaineth 0, that wryte I vnder an other lyne. Then agayne 0 in the se-

seconde place, from 0 remaineth 0, vnde[r] it I write 0 also. Next that in the thyrd place, 4 out of 2 wyll not be, therfore I add to that 2, 10, and make it 12, from that I take 4, and there resteth 8. Then saie I fa[r]there: 9 in the fourth place, and 1 (whiche I muste adde for the 10, borowed before) mak[e] 10, that must I take from 6, and bicause I canne not, I adde to the 6, 10, and then is it 16: from thense I cast 10, & there resteth 6, to bee written vnder them. Againe in the fyfte place where I fynde nothyng written, I muste set 1 for the 10 last borowed, and that 1 do I take from the 1 vnder him, and so remaynethe nought, wherfore I wryte doune a cyphar 0. Nowe haue I done wyth the subtraction: and yet in ye grosse sum remayneth 1, whiche I muste set ryght in the same place in the remayner, and so the remayner appereth to be lyke vnto the hyghest summe of the Addition, as here appeareth, wherfore I saie

206800
9400
116200
206800

that ye Additiõ was wel wrought. And thus maye youe do in any other sum of one denomination or many. Therfore nowe wyll I make any ende of Subtraction, and wyll instruct you in Multiplication.

Mul-

Multiplication is suche an operation, þ by two summes producyth the thirde: whiche thirde summe so manye tymes shall containe þ firste, as there are vnities in the second. And it serueth in the steede of many Additions. As for example. When I woulde know how many are 30 times 48. If I shoulde adde 48 thirty times it woulde be a longe work: Therfore was this worke of Multiplication diuised, which shal doe that at ones, that Addition shoulde doe at manye times.

S. I perceiue the commoditie of it partely, but I shal not see the full profite of it, till I knowe the whole vse of it. Therfore syr I beseeche you, teache me the workinge of it.

M. So I iudge it beste. but bicause that great summes can not bee multiplied but by the multiplicatiō of digets, therfore I thinke beste to shewe you fyrste the art of multiplyenge them: As when I saye, 8 times 8, or 8 times 9. &c. And as for the small digettes vnder 5, it were but follye to teache anye rule, seyng they are so easy, that euerye childe can doe it. But for the multiplication of the greater digettes, thus shall you doe.

Multiplication of digettes.

First set your digettes one ouer the other right, then loke how manye eche of them lacketh of 1 0. and wryte that againste eche of them, and that is called the Differences: as if I woulde know how many are 7 times 8, I must write those digets thus.

The difference.

Then do I looke how much 8 doth differ from 1 0, & I finde it to be 2, ÿ 2 do I write at the right hand of 8, thus.

8
7

After that I take the difference of 7 likewayes from 1 0, that is 3 and I wryte that at the right side of 7, as you see in this example.

8 2
7

Then do I drawe a lyne vnder them as in Addition, thus.

8 2
7 3

Laste of al I multiplye the two differences saying: 2 times 3 make 6, that must I euer set vnder the differences beneth the line: then must I take the one of the differences (which I will, for all is lyke) from the other diget (not from his owne) and ÿ that is lefte, must I write vnder the digits. As in this example. Yf I take 2 from 7, or thre from 8, there remaineth 5, ÿ 5 muste I write vnder ÿ digets: and then there appereth the multiplication of 7 times 8, to be 5 6. And so likewayes of anye other digettes, yf they bee aboue 5, for if they bee vnder 5, then wh[ere] th[e]

8 2
7 3
5 6

their differences bee greater then themselfe, so that they can not be taken out of them: and againe suche little summes euerye childe can multiplie, as to saie 2 times 3, or 4 times 5, and such lyke.

Scho. Truth it is. And seynge mee semeth that I understande the multiplyinge of the greater digites, I will proue by an example howe I can doe it. I woule knowe how many are 9 times 6.

M. It is all one in valewe to say, 9 times 6, or 6 times 9: but yet the ordre is beste to put the lesser summe firste, sayinge: 6 times 9. and so of all other summes.

Sc. Than wolde I knowe howe manie are 6 times 8. therfore I set the digites thus.

9
6

Then doe I set their differences at the right side: the difference of 9 which is 1, against it, and the difference of 6 which is 4, againste it also, as in this example.

9 1
6 4

And under them I drawe a line.

Then doe I multiplie the digetes togither, saying: 1 time 4 maketh 4, that do I write under the differences, thus.

9	1
6	4
	4

Then take I one of the differen[ce]s from the other digite, as one from 6, or else 4 from 9, and eche wayes there resteth 5, whyche I doe write under

 ths

the digits , and so appeareth the multiplication of 6 times 9 to bee 54. Thus I see the feate of this manner of multiplication of dygites.

Master. Nowe might you goe straighte to the multiplication of greater numbres, saue that bothe for youre ease and suertye in woorkinge, I will drawe you here a table, whereby shall appeare the multiplication of all digites, and this is it.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	56
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In whych figure whan you woulde knowe any multiplication of digits, seeke your first or laste digite in the greater figures, and from it go ryght forthe towarde the righte hande till you come vnder the digitte of youre second digite, whiche is in the higheste rowe, and than the numbre that is in the meeting of theire bothe squares, is the multiplicat that amounteth of theym. As if I woul

know

kowe by this table the multiplication of 7 times 9, seeke firste 7 in the greater figures, and than go righte forth towarde the ryghte hande, tyll you come vnder 9 of the higheste rowe, and in the meting of their squares you may se 63, which is their multiplicatiõ.

Sc. This is very good and ready. And so may I finde the multiplication of any digettes. But nowe howe shall I doe in greater summes?

M. When you wolde multiply any sum by an other, you shall marke that it is the meetest ordre to set the greatest numbre higheste, which is the place of the numbre that muste bee multiplied: and lykewayes the lesser numbre vnder it, for that is the place of the Multiplier or multiplicatour, that is to saye the numbre by whiche multiplication is made and is in Englishe alwaies putte before thys worde, Tymes: in suche speakyng when I saye 20 times 70. And the numbre that foloweth this woorde Tymes, is that whyche muste be multiplied.

To multiplie greater summes

Multiplier

Therfore when I woulde multiplye one numbre by an other, I must write the greatest, highest, and the lesser vnder it, as in additiõ. And vnder them muste I drawe a line. As for example. If I woulde multiply 264 by 29 I muste set them thus.

$$\begin{array}{r} 264 \\ 29 \\ \hline \end{array}$$

Than muste I multiply euery figure of the higher row, by euery

 figure

fygure of the nether rowe: and that that amounteth, I muste sette vnder the lyne, as thus: Firste I do multiplye 4 by 9, saying: 9 times 4 (or 4 times 9, whiche is a lone) & ye maketh 36, as the table before of digets doth declare. of that 36 I must write ye 6 that is the digete vnder the 9, and the 3 in the nexte place toward the lefte hande.

264
29
———
36

Then come I to the ij. figure, of ye hyther row & say: 9 times 6 make 54, of which I write ye 4 vnder the 3, & the 5 vnder ye nexte place (as ye reasō willeth me) thus.

264
29
———
536
4

After that come I to the nexte figure, whiche is 2, and do multiplie it by 9, and that maketh 18: whereof I write 8 vnder ye thirde place, and the article 1 in the fourth place, thus.

264
29
———
1536
84

And so haue I ended the fyrste figure of the multiplier. Then begin I wyth the nexte fygure, and multiply it into all the higher fygures, as thus.

294
29
———
1536
84
8

Firste 2 tymes 4, make 8, that do I write vnder the seconde place, for euermore the digite or fyrste figure of the multiplication that amounteth of the firste fygure of the higher number muste be set vnder the multiplier of it, and th

the other in theire ordre toward the left hād.

Scholar. I vnderstande you thus, that the digitte of the summe amountinge of the multiplication of t he firste fygure of the higher rowe, by the first fygure of the lower row: or multiplier, muste bee sette vnder the firste place: and that that amounteth of the same first fygure by the seconde multiplyer, muste be set vnder the seconde place: and so of the other, if there be more multipliers.

Ma. So meane I in deede: And, if there amount but a digitte, then must yt bee sette vnder the multiplier. And nowe to go forth: I multiply by the same 2, the seconde figure of the higher rowe, whiche is 6, sayinge: 2 tymes 6 make 12, wherof I write the dygit 2 vnder the third place, and ÿ article 1, I write vnder the fourthe place.

```
  264
   29
 ----
 1536
 184
 28
```

Then do I multiply the laste figure of the higher summe, by that same 2: saying: 2 times 2 is 4, whiche I wryte vnder the fourthe place. And so haue I ended the whole multiplication: And the summes stande thus.

```
  264
   29
 ----
 1536
 184
 428
```

Than muste I drawe a lyne vnder all those summes that amounte of the multiplication: and muste adde all them into one summe, as in thys example

you may see.

Where in the first place I finde but 6, & therfore write I it vnder the lyne. Then in the second place 8, 4, 3, make 15, wherof I write 5, & kepe 1 in my mind. and so forth as you learned in addition. And so appereth yͤ whole summe to be 7656, which amounteth of the multiplication of 264 by 29.

```
 264
  29
----
1536
 184
 428
----
7656
```

Scho. If there be no more to bee obserued in it, then can I do it I suppose. as by this example I shall proue. I woulde multiply 1365 by 236 wherfore I set theym thus.

```
1365
 236
----
```

Than do I multiplye 5 by 6, sayinge: 6 times 5 make 30, of whiche I write yͤ cyphre in the first place, and the article 3 in the seconde place.

```
1365
 236
----
  30
```

Than do I by the same 6, multiplye the second figure of the higher sum which is 6, saying: 6 times 6, make 36: of which I write the 6 vnder the seconde place, and the 3 vnder the thirde place.

```
1365
 236
----
 330
  6
```

Than do I multiply the third figure whiche is 3 by the same 6, and that maketh 18, of that I set the 8 vnder the third place & the 1 in the fourth place.

```
1365
 236
----
1330
 86
```

Than

than come I to the laste figure of the higher summe, & multiply it by 6, saying: 6 times 1 make 6, that do I write vnder þ fourth place. And so haue I ended the firste multiplier.

```
 1365
  236
 ----
 1330
686
```

Than begin I with the secõd multiplier, and saye firste: 3 times 5, that maketh 15, of whiche I set the 5 vnder þ seconde place, because þ the multiplier is there, & þ 1 I set vnder the third place.

```
 1365
  236
 ----
 1330
686
 15
```

Than come I to the seconde figure þ is 6, and multiply it by 3 which maketh 18, of which I set the 8 vnder þ third place, & the article 1 in þ fourth place.

```
 1365
  236
 ----
 1330
686
115
8
```

Than come I to the thirde fygure whiche is 3, & multiplye it by 3, saing: 3 times 3 make 9, whiche bicause it is but one dygette, I sette vnder the fourthe place.

```
 1365
  236
 ----
 1330
686
115
98
```

And then commyng to þ laste figure 1, I multiply it by 3, and maketh 3, whych I sette in the fyfte place, and than haue I ended two of the multipliers, & the summes stande thus.

```
  1365
   236
  ----
  1330
 686
 115
398
```

1365
236
1330
686
115
398
10

Then come I to the thirde multiplier, and multiply it into euerye fygure of the higher sum, & fyrste I say: two times 5 makes 10, of which I set ỷ ciphre vnder ỷ multiplier in the thirde place, & ỷ article 1 in the fourth place.

1365
236
1330
686
115
398
110
2

And so multiplyinge the seconde figure 6 by that same 2, there amounteth 12: where of I write the digitte 2 vnder ỷ fourth place, and ỷ article 1 vnder in the fifte place.

1365
236
1330
686
115
898
110
362

Nowe doo I multyplie by the thirde figure of the hygher summe, whyche is 3, and that maketh 6: whiche I set vnder the fift place.

Than come I to the laste place, and multiplie that 1 by 2, and there amounteth 2, whiche I set in the syxte place, and then doothe the summes stande thus.

1365
236
1330
686
115
398
110
262
322140

And so haue I ended the whole multiplicatiõ.

But nowe (as you taught me) to knowe

what this whole summe is, I muste adde all those parcels togyther, and than vnder the lyne wyll appeare as you see, the grosse or totall summe, that is 322140.

Maister. That is wel doone.

Scholer. Than me thinketh I woulde call it well doone, when I knewe whether I had well done or no.

Master. It may be treid by 9, as addition was, but the surest proofe is by diuisiō, and therfore I wyll reserue that tyll you haue learned the arte of Diuision.

And before wee passe from Multiplicatiō, I wyll yet shewe you an other manner of Multiplication, which ar counted of somme men both more ready and more certaine, of whiche the one differeth nothinge from this that I haue taught you, saue that it doothe vnderstande alwaies the articles, and ioyne theym to the nexte summe: and therfore I wyll declare it onely by an example. An other manner of Multiplication.

If I woulde multiply 1542, by 365. I muste sette theim as I sayde before, and then doo I multiply 2 by 5. and it maketh 10, of whiche I write the article vnder the fyrste place, and keepe the digite 1 in my minde.

Than saye I forthe: 5 times 4 doo make 20, and the 1 in my minde are 21, therof I write the 1 vnder the seconde place, and kepe the 2 in

1542	
365	
0	

in my minde.

1542
365
10

Than come I to the third figure 5, saynge: 5 tymes 5 make 25, and the 2 in my mynde, make 27, werof I wryte the 7 vnder the thirde place, and kepe the article 2 in my minde.

1542
365
710

Thã comminge to the laste figure I say: 5 tymes 1 make 5, and 2 in my mynde make 7, that dooe I wryte vnder the fourth place

1542
365
7710

And than haue I ended my fyrste multiplyer.

Than do I likwaies with the seconde multiplier, saying: 6 times 2 make 12, therof I wryte the digitte 2 vnder the seconde place, and keepe the article 1 in my minde.

1542
365
7710
2

Than saye I forthe: 6 tymes 4 maketh 24, and 1 in my minde make 25, so I set ẏ 5 vnder the thyrde place, & keepe the 2 in my mynde.

1542
365
7710
52

Than multiplie I forth, saying: 6 times 5 maketh 30, & 2 in minde make 32, wherof I write the 2 vnder the iiij. place, and kepe the 3 in my mynde.

1542
365
7710
252

Than do I multiplie the laste figure

figure 1 by 6, & it maketh 6, to that I adde the 3 in my minde, and it maketh 9, which I write in the fyfte place.

1542
365
7710
9252

And so haue I ended 2 figures of the multiplier.

Than with the third and last multiplier do I likewaies, & say fyrst: 3 times 2, make 6, which I write in the third place vnder the multiplier.

1452
365
7710
9252
6

Than by that 3 do I multyplye likewaies the seconde fygure 4, & it maketh 12, where of I write the diget 2 vnder the fourth place, and the article 1 I keepe in minde.

1542
365
7710
9252
26

Than come I to the thirde figure 5, sayinge: 3 times 5, maketh 15, and the 1 in my minde make 16, therof I write the 6 vnder the fifte place, and keepe the article 1 in my minde.

1542
365
7710
9252
626

Than come I to the laste figure whyche is 1, and multiply it by 3, and it maketh 3, therto I adde the one in my minde, & it maketh 4: whiche I write in the 6 place. And than haue I ended the multiplication, and the figures stande in ordre thus.

1542
365
7710
9252
4626

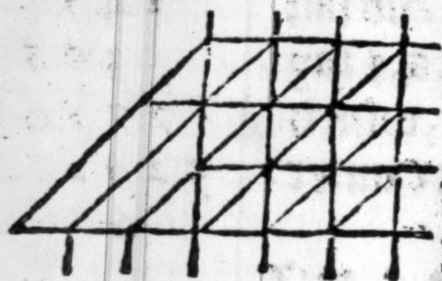
Whiche

Whiche parcels yf I adde into one sum, it wyll be 5 6 2 8 3 0, which is the grosse or totall sum of all that multiplication.

Sc. Wel, this manner of Multiplication I perceaue: but what other sorts haue you?

An other waie of multiplication.

M. Ther is one way that is wroughte by a checker table, made thus.



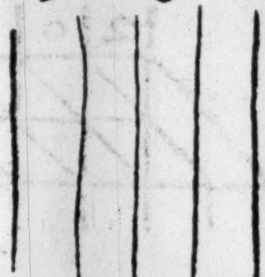
Loke howe manye places youre sum hathe ẏ you woulde multiplie, so manye squares muste you make in your table, from the righte side to the lefte, and so manye from the higher parte to the lower, as ther be places in your multiplier. Than sette downe youre greateste summe firste on the toppe of the table, euerye figure in due ordre, in a square alone, I meane in those squares that bee open and vncrossed. And lykewaies in those like squares at the right hande, sette doune youre multyplicatour or multiplier, the laste figure in the highest place, & so dounwarde, that the firste figure may be in the lowest place.

Sc. Syr if it please you, me thinketh that I vnderstande you beste, whan you doe not stande longe in tellinge the rule before examples. But propose some example, and then in declaringe it, bringe in the rules withall.

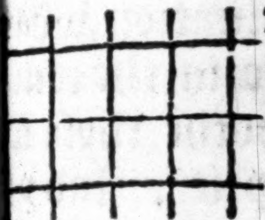
M. In dede, ẏ way is easiest for a yong learner, therfore wyll I euen so do. Take this example.

exãple: now I wold multiply 2036 by 25.

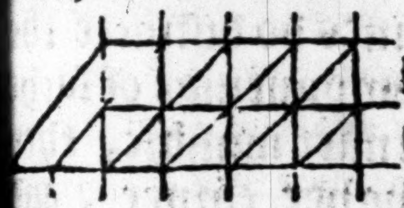
Firste I consider that my greatest numbre hathe foure figures or places, & therfore I make so many roumes betwene lines, thus.



Than I see that of my multipliers ther ar two, wherfore I drawe so manye lines a crosse ye other, that there may bee two roumes betwene them, thus.

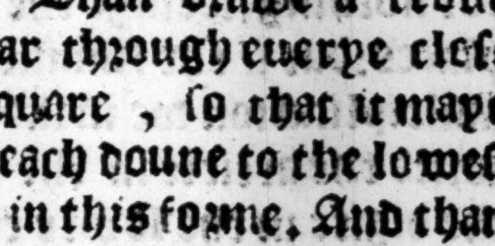


But you muste not forget to let the endes of the lines runne out, as it appeareth in this Patron, for in those open squares muste youre two firste numbres and all the totall summe be set.



Than drawe a crosse bar through euerye close square, so that it maye reach doune to the lowest ouerthwarte line, as in this forme. And than is your checker forme prepared.

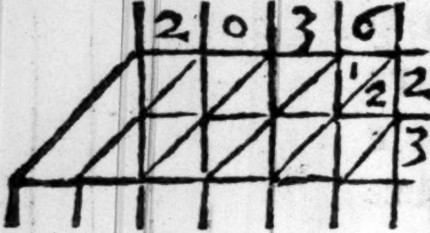
Than sette downe your first or greateste summe on the toppe, & your multiplier on the righte side in the open squares, thus.



Than begin to multiplie the firste figure of the higher summe by the higheste of the multiplier, saying: 2 times 6 make 12, that

12 muste

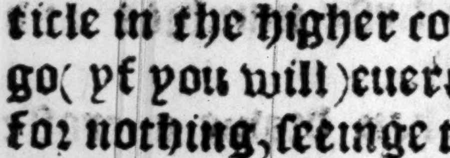
12 muste you write in the square that is a-



gainste the 2 and the 6, but of suche maner that the diget be set in the nether corner of the square & the article in ye higher corner, as you may see in this example

And so of euery other multiplication, what euer amounteth you muste write in the common square, which is againste bothe those figures, by whiche you do multiplie. And yf that summe do make but one digitte, than must it be set in the lower corner of ye square, but yf it make an article, then write the article in the higher corner, and let the cyphre go (yf you will) euermore: for here it serueth for nothing, seeinge the lines do distincte the places: but if the summe amountinge of suche multiplication do make a mixt numbre, then write the article in the higher corner, and the digite in the lower corner, as I dyd by that 12.

Then whan you haue multiplied and ended the fyrste fygure, come to the nexte, and multiplye it in like manner, as in sayinge



2 times 3 is 6, that 6 bicause it is but a digit you shall sette in the nether corner of ye square next vnder 3, thus.

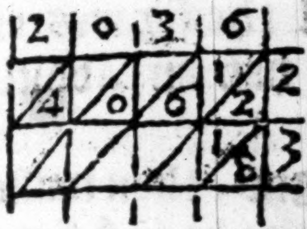
Than go forthe saying: 2 times 0 is 0, se

tha

that vnder the barre (if you lyst) in the third square.

Then forthe and saye: 2 tymes 2 make 4, that set in the last square vnder ẏ bar, so haue you ended the first multiplier.

Come now to the second multiplier, and saye: 3 times 6 make 18, of which summe

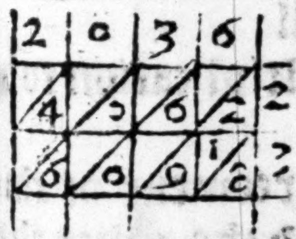


the article 1 must be sette aboue the barre, in that square that is next to the 3 (as you see here) and the 8 vnder the barre.

Then saye: 3 times 3 maketh 9, set it in the next square beneth the barre. Then 3 times 0 is 0, write it in the next square, or let it go, for all is one.

Sco. I perceiue it well: for here the lines distincte ẏ places wherfore cyphres do onely serue, and therefore here they nede not to bee.

M. Then saye further: 3 times 2 make 6: write that in the last square,



then will the whole figure stande thus.

Sc. Now could I (me semeth) doe like againe: but howe shall I doe nowe to gather the sum?

Mast. Marke firste the order of the places in this figure, and so shall you perceaue the reason of gathering them into a summe.

The sloape barres doe parte the places, so that the firste place is the lowest corner (in all such figures) of the nether most square next the right hand: and all the halfe squares betwene that barre and the nexte, standeth for the seconde place, and so the roume betwene that and the next barre is the thirde place: and so forthe. Nowe if you perceiue this, then muste you adde all the figures of one place together, as if you had an Addition of diuers summes.

S. If I vnderstand you right, then muste I take here in this example 8 to be in ẏ firste place, 9, 1 and 2 in the second: 0, 6, 1, in the thirde: 6, 0, in the fourth: 4 in the fifte: and the sixt place hath no figure.

Ma. You say well, & the reason is because the multiplication seruinge to that square, made but a digetre.

Scho. Then is it all one, as if they stoode thus.

			1	2
		0	6	1
4	6	0	0	8

Ma. Euen so it is: and nowe adde this summe, and ther wil appeare the totall of the multiplication to be 4 6 8 2 8.

A proofe without squares.

And if you will see the agreement of this maner of multiplication, and the other that you learned before, then multiplie those two summes (that is 2 0 3 6, by 2 3) after the first maner without squares.

2	0	3	6
		2	3

S. Yo

S. You mean to set them thus in ordre.

And thē multiplie 3 into 6, make 18 : 3 times 3 make 9, 3 times 0 is 0. then 3 times 2 make 6, whiche must be set thus.

2	0	3	6
		2	3
		1	8
6	0	9	

Then doe I lykwayes with the second multiplier, sayenge: 2 times 6 make 12, 2 times 3 make 6, 2 times 0 is 0: and 2 times 2 make 4, which when I adde to the other, then will the whole multiplication stande thus.

Maist. So that you may see in euery place the same figures, as they were in the multiplication by squares, though they differ in height and lownesse of places, but beynge added together, they make one summe.

	2	0	3	6
			2	3
			1	8
	6	0	9	
1	1	1	2	
4	6	8	2	8

And thus nowe ye haue learned thre sortes of multiplication, which you liketh beste, that may you vse.

Yet are there other formes, but sith they nothing differ from these thre in effecte, but onely in setting of the numbres, I will ouerpasse them tyll a more meeter place and time. And nowe will I instructe you in Diuision, so that you thinke youre selfe sufficientlye to perceiue what I haue taught you.

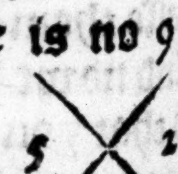
Scholer. Yes sir I thanke you: but I doe not perceiue howe to examine my woorke, to try whether I haue well done or no.

M. That is commonlye vsed by the proofe of 9, as you learned before in Addition and Subtraction, saue that it hath this waies diuers from them.

First you must make a crosse after this manner.

Then muste you examine youre summe that shoulde be multiplied, and looke what remaineth after, casting away of 9, that set you at the one side of the crosse: then examine the multiplier, and whatsouer remaineth in it, after castinge away 9 as often as you can, write that at the other side of the crosse, then must you multiply those two numbres together, & looke what amounteth therof, if it be vnder 9, write it at the higher parte of the crosse: but if it bee aboue 9, then take thence 9 as often as you can, and write the reste at the head of the crosse: As in the laste example of multiplication, the numbre to be multiplied is 2 0 3 6, wherin is ones 9, & 2 remayneth, which I write at one syde of a crosse, thus.

Than doe I examine the multiplier, which is 2 3, wherin ther is no 9, but 5 in all, that 5 therfore I set at the other syde of the crosse, thus.



Then doe I multiplie 5 by 2, and it maketh 1 0, from which I withdrawe 9, and there resteth 1, that 1 do I set at the head o

th

the crosse, than do I examine the grosse sum amounting of the multiplication, whiche is 4 6 8 2 8, wherein I finde 9 thre times & 1 remaining, yͤ 1 I set at the foote of yͤ crosse: & then I se it to agre with the other 1 at the top of yͤ crosse, & so knowe I yͭ I haue done well: for if they two did differ, than were my worke vaine, and the multiplication false.

This is the common proofe, but the moste certaine proofe is by Diuision, of whiche I wyll anon instructe you.

Sch. Syr what is the chiefe vse of Multiplication?

M. The vse of it is greater then you can yet vnderstande: howbeit, these plaine commodities it hath, that if you woulde resolue any great & whole valure into many smale & lesse portions as if you woulde chaunge poundes into shillings, pennies or anye other greater or smaller parcels, by multiplication, ye shall do it spedely & easylye. Also if you should nede to adde one sum to it selfe, or to any other oftentimes, you shall doe it by Multiplication much more spedely, readily, easylye & surely, than by often and sundrye Additions. Take you these commodities grosselye shewed for an aunswere at this tyme, and hereafter I will more abundantlye make you to perceiue the vse of it.

Scholar.

Well syr, then in Diuision I pray you to instructe mee. But mee thinketh by the name of it, that it should be al one wt Multiplication: for I call that Diuision, when anye thinge is parted into diuers and many partes.

M. You take it as it is taken commonlye, howbeit, if you marke wel, you shall perceiue that it is quite contrarye to Multiplication, and doth not parte one thinge or fewe things into manye, but contrarye waies it bringeth many parcels into few: but yet so, that these fewe taken together, are equall in valure to the other many. for by Diuision pennies are tourned into shillinges, and shillinges into poundes. as for example of 1 2 0 shillings, it maketh 6 pounde. so are 1 2 0 tourned into 6, whiche is a smaller numbre: but then if you consider the denominatours, you shall see that they are such, that one of the latter is equall to 2 0 of the firste, and so in valewe the summes are one, though in numbre they be farre differ, and the latter summe is the lesser, and so is it alwaies in diuision, howbeit, yet in ye working the summe, is parted by

Quotient number.

by an other, and therof doth it take the name.

Sc. I thinke I shall better understand the reason of the name, when I knowe the use of the worke, therfore nowe would I gladly learne that.

Ma. Diuision is a particion of a greater summe by a lesser. Therefore (as you maye perceiue) unto Diuision are required twoo numbres: the firste, whiche shoulde be diuided, and that muste be the greater: and the seconde, by whiche the other muste be diuided, and that is the lesser, and is called the Diuisor. Diuision what it is

The firste must be firste written, and the second so set under it, that the laste figure of the lower number be right under the laste of the higher, contrarie waies to the worke of the other kindes of Arithmetike: for in the the two fyrste figures were sette euer meete one under the other, but in Diuision the laste figures muste be set meete, except it chaunce so that the laste figure of the Diuisor be greater then the laste of the higher number, for then you shall see the laste of the Diuiseur under the laste (saue one) of the higher numbre: as for example.

If you should diuide 3 6 5 (which are the summe of the daies of a yere) by 2 8, which are the daies of a cōmon moneth, then should you set them thus.

3 6 5
2 8

But if you would diuide those 3 6 5 daies,

 by

by 5 2, which is the number of wee-
kes in one yere, then should you set
them thus.

3 6 5
5 2

Lykewayes if I woulde diuide
the same 3 6 5 by 4, which is
the sum of ẏ quarters of a yere,
then must I set them thus.

3 6 5
4

Scholar. Syr this doe I vnderstand, but
howe nowe shoulde I doe to diuide the one
by the other?

M. You must begin with the laste figure
next the left hande, and see howe many times
the laste figure of the diuisor, maie bee ta-
ken out of the laste figure of the ouer num-
bre, & that shall you note within a
croked lyne towarde your right
hand. As for example.
I would diuide 3 6 5 by 2 8, the
set I those two summes thus.

3 6 5 (
2 8

And I looke howe many tymes I maye
finde 2 (which is the laste figure of the diui-
sour) in 3, (which is the laste of the numbre
to be diuided) and consideringe that I can
take 2 out of three but ones, I make a croked
line at the right hand of the numbres, & with-
in it I sette 1, and that is called the Quo-
tiant numbre: Then bicause that when 2 is
taken out of 3 there remaineth 1,
I must write that 1 ouer 3, and
deface or cancell the 3 and the 2,
the will the figures stande thus.

Quotient numbe r.

1
~~3~~ 6 5 (1
~~2~~

Then

Then must I go in the next figure of the diuisor, and take it likewiaes so many times out of the figures that be ouer it, and looke what doeth remaine, that I must write ouer them, and cancell them, as in this example.

I take ones 8 out of 16, and there remaineth 8, which I must set ouer the 6, and cancell or crosse out the 16, and ÿ 8 of the diuisor: And thē will the figures stād thus.

$$\begin{array}{lll} \cancel{1} & 8 & \\ \cancel{1} & \cancel{6} & 5 \;(1 \\ \cancel{1} & 8 & \end{array}$$

And so haue I ones wrought.

Scho. So I perceiue that you take the nether figure not onely out of the other that is right ouer him, but out of that with the other also that remaineth before, and are written towarde the left hande.

Ma. So must you do, for you must so take the diuisor out of the ouer number, that there remaine not ouer it so great a summe as yt selfe is: for then were your worke in vayne.

But yet againe here must you marke, that when you seeke howe many times the laste figure of the diuisour may be founde in the numbre ouer him, that you looke also whether you may as often finde all the figures following in those ÿ are aboue them, if not, take your Quotiente lesse by one, and then proue again, & so still, til you finde a mete Quotient.

When you haue thus wrought ones, then must you begin againe, and write your diui-

 sour

four a newe, nerer towarde the right hand by one place, as in this example, you shall set 2 vnder 8, & 8 vnder 5, thus.

~~2~~	8	
~~3~~	6	5 (2
~~2~~ ~~8~~	8	
	2	

Then (as before) seeke howe many times you may take the last diuisour out of the numbre ouer him.

Scho. That may I do here 4 times.

Mast. Trueth it is that you may finde 2 foure times in 8: but than marke whether you can finde the figure folowing so many times in the other that is ouer him. Can you finde 8 foure times in 5?

Scholer. No, nother yet ones.

Ma. Therfore take 2 out of 8, ones lesse.

Scholer. That is 3 times.

M. Wel, thē 3 times 2 make 6: if I take 6 out of 8, ther remaineth 2: which 2 with the 5 folowing make 2 5, in which sum I mai find 8 iij. times also, and therfore I take 3 as a true quotient, and write it within the croked line of the quotient, before the one, thus.

~~2~~	8	
~~3~~ ~~6~~	5	(1 3
~~2~~ ~~8~~	8	
	2	

Then say I: 3 times 2 make 6: thē 6 out of 8, resteth 2, therefore I cancell the 2 and the 8, and write ouer it the 2 that doeth remaine, thus.

	2	
~~2~~	8	
~~3~~ ~~6~~	~~5~~	(1 3
~~2~~ ~~8~~	8	
	~~2~~	

Then

Then do I take 8 as ma-
ny times out of 2 5, sayinge:
3 times 8 make 2 4: and if I
take 2 4 out of 2 5, ther re-
maineth 1, so then I cãcel 2 5
& 8, & ouer ỹ 5 I set 1, thus.

1
2 8 1
2 0 5 (1 3
5 8 8
2

And nowe haue I doon with diuidinge, for I canne finde my diuisour 2 8 no more in the ouer summe.

Scholer. No, except you would part the 1 that remaineth, into 2 8 partes.

Maister. That is well sayde, and so muste we do in such cases, when there remaineth anye thinge: but I will let that passe nowe, and will make you perfect in whole diuision, and will hereafter teache you peculiarlye of broken numbre, called Fractions.

Nowe yf you do perceiue the ordre of Diuision, than dooe you diuide this summe 1 3 6 2 8 0 by 4 5 2.

Sch. Fyrste I set downe the numbre that should be diuided, than do I set the diuisour vnder it, so that the last fygure of it be righte vnder the laste fygure of the ouer numbre: Than will it be thus.

1 3 6 2 8 0
4 5 2

M. Can you take the laste of your diuisor (which is 4) out of 1, whiche ys the laste of the ouer numbre?

Sc. I had forgotten, bicause the laste of the diuisor can not bee taken out of the laste of the ouer numbre, in as muche as it is ỹ greater:

therfore

therefore muste I sette ÿ diuisor one place more forwarde towarde the ryght hande, thus.

1 3 6 2 8 0
4 5 2

And then muste I looke howe often I maye finde the laste fygure of the diuisor (that is 4) in 1 3, whiche thing I may do 3 times, therfore do I saye: 3 times 4 is 1 2, which I take out of 1 3, and there remaineth 1. Then do I make at the ryght hande of my summes a crooked lyne, and write before it my quotiente 3, and I cancell 1 3 and 4, and ouer the 3 I sette the 1 that remaineth, and than the fygures stande thus.

1
~~1~~ ~~3~~ 6 2 8 0 (3
4 5 2

Than do I multiplye the same quotient into euerye fygure of the diuisor, and withdrawe the summe that amounteth out of the numbres ouer theim, as firste I saie: 3 times 5, make 1 5, which I take from 1 6, & there resteth 1: I cancell therefore 1 6 & 5 and write ouer the 6 ÿ one ÿ remaineth, thus.

~~1~~ 1
~~1~~ ~~3~~ 6 2 6 0 (4
4 ~~5~~ 2

Than do I say like waies 3 times 2 make 6, which I tak out of 1 2, and there resteth 6, therefore I cancel the 1 2 & the 2, & ouer the 2 I write 6 that remaineth, thus.

~~1~~ ~~1~~ 6
~~1~~ ~~3~~ ~~5~~ ~~2~~ 8 0 (3
4 ~~5~~ ~~2~~

Then shoulde I set forwarde the diuisor

into

into the next place toward the right hand, thus.

```
    x x 6
x 3 5 x 8 0 (3
    4 5 x
      4
```

M. But you may see, þ ouer the 4 is noe fygure, therfore muste I set the diuisor yet forewarder by an other place.

And marke whan so euer it chaunseth so, that you shoulde set forwarde the diuisor, and that it can not stande there, bicause there ys no numbre ouer the laste place, or if there bee any, it is lesser then the laste fygure of the dyuisor, than muste you remoue the diuisor yet ones againe: and bycause that hys fyrste place of remouing fayled hym, therefore shall you write in the quotient a cyphre o. And if you shuld by chance need to do so often times, for euery tyme write a cyphre in the quotient. The reason of thys, will I shewe hereafter.

```
      x x 6
  x 3 5 x 8 0
4 5 x 5 2   (3 0
      4
```

Scholer. Then muste I sette my summes thus.

And bicause I remoued þ diuisor, so that I ouerskipped one place, I must wryte a cyphre in the quotient: & than muste I seeke a newe quotient: as in this example I must saie, how many times 4 is there in 6? and syth it canne be but ones, therfore

```
        2
    x x 6
x 3 6 2 8 0 (40
4 5 x 5 2
      4
```

not

do I write 1 in the quoti- ent, and then saye I: 1 tymes 4 take out of 6, remaineth 2, I cancel ye 6 and the 4, & write 2 over them, thus.

2
~~1~~ ~~1~~ ~~6~~
~~1~~ ~~3~~ 6 2 8 0 (3 0 1
4 ~~5~~ ~~2~~ 5 2
4

Then say I againe: ones 5 out of 2 8, remaineth 2 3: I let ye 2 stand as it did, & over ye 8 I set 3, canceling ye 8 & ye 5 under it, thus

2
~~1~~ ~~1~~ ~~6~~ 3
~~1~~ ~~3~~ ~~6~~ ~~2~~ 8 0 (3 0 1
4 ~~5~~ ~~2~~ ~~5~~ 2
4

M. You might as wel haue said: ones 5. out of 8, & so remayneth 3, but now go foorth.

Scholer. Than ones 2 out of 0, can not bee, what shall I now doo?

M. Borow of the next numbre that is behynd (for there is 2 3 0) and doe as you learned in Subtraction in a like case.

Scholar. Than muste I borrowe 1 of the 3 comming behinde nexte, and make that 0 to bee 1 0, and then take I 2 out of 1 0, & there resteth 8. And bicause I borowed one of the 3, I must cancell the 3, & write 2 over it, than doth the figure stand thus.

2 2
~~1~~ ~~1~~ ~~6~~ ~~3~~ 8
~~1~~ ~~3~~ ~~6~~ ~~2~~ 8 ~~0~~ (3 0 1
4 ~~5~~ ~~2~~ ~~5~~ ~~2~~
4

M. Nowe haue you doone; and yet remayneth 2 2 8, & your quotient dooth shew you ye if you diuide 1 3 6 2 8 0 by 4 5 2, you shall finde

youre

your diuisor in your greater numbre 3 0 1, ꝥ is CCC. times, & ones 2 2 8 remaining.

And in the other example, where I diuided 3 6 5 by 2 8, the quotiēt was 1 3, & 1 remained: wherby I knowe that in a yeare (which conteyneth 3 6 5 daies) there are 1 3 monethes, reckening 2 8 daies (or 4 wekes) iuste to a moneth, and 1 daie more.

Scholer. Why then doe we call a yeare but 1 2 monethes?

M. Of that at a more conuenient time wil I fullie instructe you: but nowe it is not conuenient to entangle your minde with other thinges, then doe directlie perteine to your matter. Therfore if you can remembre what you haue hearde, you haue learned a shorte maner of diuision, whiche I woulde haue you often to practise, so that you maye bee perfecte in it, and hereafter I wyll shew you certaine other proper pointes touching it.

S. Then I pray you, yet tell me howe I shall examine and trie my woorke whether I haue done wel or no, that thoughe no man be by me to tell me, yet I maie perceaue it my selfe.

M. Some men (yea and commonly) do trie that by the rule of 9, as in all the other kindes, saue that their order is this. Firste they cast awaie 9, as often as they can out of the diuisor, and that that remaineth, they set at one side of a crosse. As in our firste example, Profe.

the

the diuisor was 2 8, from which you may take 9 three times and 1 remaineth, which they set by a crosse, thus.

Then doe they likewise examine the quotient (which in our example is 1 3) and from thence they cast away 9 as often as they can, and the remainer they set at the other side of the crosse, and then multiplye they together those 2 remainers: and to it that amounteth, they adde the remainer of the diuision, if there were any, from that whole summe they withdrawe 9, as often as they can, and the reste they set at the head of the crosse. as in our example the quotient is 1 3, which maketh onely 4, and therefore muste you sette 4 at the other side of the crosse, thus.

Then multiply 4 by 1, and it yeldeth but 4, therto adde the remainer of the diuision (which was 1) and it will be 5, which summe doeth not amounte to 9, and therefore muste bee sette wholly at the headde of the crosse, as you see here.

And this number on the heade of the crosse is the first proofe, to which yf you finde an other like in the number that was diuided, then haue you doone well.

Therfore now shall you likewise examine the whole summe that was diuided, and take

awa

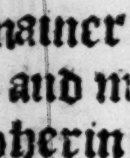
alwaie 9, as often as you can, and that that remaineth, set at the foote of the crosse: and if it be equal to that in the heade of the crosse, then haue you well done, else not.

As in our example the whole summe was 3, 6, 5, which maketh 14, from that take 9, and there resteth 5, which set at the foote of the crosse, thus, and you shal see that they agree: therfore haue you well done.

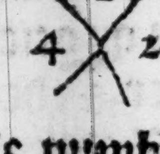
S. Nowe will I likewise examine our second example, wher the diuisor was 4, 5, 2 which maketh 11, from thence I take 9, & the 2 that remaineth I set at ye right side of a crosse, thus. An other proofe.

Then examine I the quotient, whiche was 3, 0, 1, wher I find but onli 4, that doo I set at the other side of the crosse, thus.

Then do I multiplie 4 by 2, and it maketh 8, to that doo I adde the Remainer of the diuision, (which was 2, 2, 8, and maketh 12) and they two make 20, wherin I finde twyse 9, and 2 remaining, that 2 must I set at the heade of the crosse, thus.



Then doo I examine the whole numbre to be deuided, which was 136280, wher I finde twise 9, and 2 remaining, which I set at the foote of the crosse, thus.



And bicause that it doeth agree with the figure at the heade of the crosse, I knowe that the diuision was well wrought.

M. This is the common profe: howbeit, the moore certaine working is by the contrarie kinde, as to proue diuision by multiplication, thus.

A proofe by Multipilcatiō.

Multiplie the quotient by the Diuisour, and if the summe that amounteth bee equall to the summe that shoulde be deuided, then haue you well diuided, else not.

Howebeit, this muste you marke, that if there remained any thing after the diuision, that must you adde to the summe that amounteth of the multiplication: as in our firste example the quotient was 1 3, and the diuisor was 2 8. Now multipli the one by the other and the summe will bee 3 6 4: to that if you adde the one that remaineth after ỿ diuision, then will it be 3 6 5, which was the summe that should be diuided, and therfore I knowe that I haue well done.

S. Now wil I proue the same in the seconde example, whose diuisor was 4 5 2, and the quotient 3 6 1: these do I multiplie togther, & ther amoūteth 1 3 6 0 5 2, to which if I adde the 2 2 8 that remaineth, then will it be 1 3 6 2 8 0, which was the whole summe to be diuided, and therfore I perceiue that I haue well done.

M. This is the syrest way to examine Diuisi-

uision by multiplication;and contrary waies ÿ surest profe of Multiplication, is Diuision.

And therfore now will I shewe how you may proue Multiplication by Diuision.

When you haue ended Multiplication, and woulde knowe whether you haue well done or not, set the grosse sum that amounteth of the Multiplication ouermoste, and diuide it by the multiplier: and if the quotient be the same numbre that shoulde bee multiplied, then haue you wel wrought, else not: as in that example where we multiplied 264 by 29, the grosse summe was 7656.

A profe of Multiplication by Diuision.

Nowe if you wyll knowe whether that Multiplication be true, you shall diuide that 7656 by ÿ Multiplier 29: and you shall perceaue that the quotiente will bee 264, and that is a token that you haue well wrought.

S. By youre pacience I will proue that: and first I set downe the grosse summe and the Multiplier, not after the rule of Multiplication, but after the rule of Diuision. for now that numbre is become the diuisore that was before the multiplier, I shall set theim therfore thus.

$$\begin{array}{r} 7656 \\ 29 \end{array}$$

Then shall I seeke howe many times 2 in 7, that may bee 3 tymes, and 1 remaineth: but then may not 9 be found so often in 16. therfore must I take a lesser quotient, that is to say, 2: then say I

twise 2 maketh 4, which I take out of 7, & there remaineth 3. thē do I cancel 7 and 2, and ouer 8 I write 3, and in the quotient I set 2. so the figures stande thus.

3
~~7~~ 6 5 6 (2
~~2~~ 9

Thē say I forthe, 2 times 9 make 18, which I bate oute of 36, and there resteth 18, thē cancel I 3, and ouer hym set 1: & likewise I cancel 6 and 9, and ouer them I sette 8, so that thus stande the fygures.

1
~~3~~ 8
~~7~~ ~~6~~ 5 6 (2
~~2~~ ~~9~~

Then doe I set forwarde the diuisor by one place, and seeke a newe quotient, that is to say, how many times 2 are in 18, which I finde to be 9 times, but then can I not finde 9 so many times in 5, therfore I take a lesser quotient, as to say, 8, but yet is that to great, for if I take 8 times 2, oute of 18, ther remaineth but 2: and I can not finde 8 times 9 in 25, therfore yet I take a lesser quotient, ỹ is 7, which is also to great. for if I take 7 times 2 out of 18, ther resteth 4, but now I can not take 7 times 9 out of 45, therfore yet I seeke a lesser quotient, as to say, 6, then say I, 6 times 2 make 12, that I take out of 18 and there remaineth 6: so I cancel that 18 and the 2, and write 6 ouer 8, thus.

~~1~~ 6
~~3~~ 8
~~7~~ ~~6~~ 5 6 (2 6
~~2~~ ~~9~~ 9
~~2~~

The

Then say I forth: 6 times 9 maketh 5 4, that take I out of 6 5, & there remaineth 1 1, and the figures stande thus:

~~1~~ 6
~~3~~ ~~8~~ 1
~~7~~ ~~6~~ ~~5~~ 6 (26
~~2~~ ~~9~~ ~~9~~
~~2~~

Then must I set forth the diuisor againe & seke a newe quotient, whiche will be 4, for though I maie finde 2 in 1 1 fiue times, & 1 remaine, yet I can not find 9 so often in 1 6, therfore I set ÿ figures thus.

1
~~1~~ ~~6~~
~~3~~ ~~8~~ 1
~~7~~ ~~6~~ ~~5~~ 6 (2 6 4
~~2~~ ~~9~~ ~~9~~ 9
~~2~~ 2

And the 4 in ÿ quotient I multiplye into the figures of the diuisour, saying: 4 times 2 maketh 8, which I tak out of 1 1, & ther resteth 3, therfore I cãcel the 1 1 and the 2, and set 3 ouer the first place of 1 1, thus.

~~1~~
~~1~~ ~~6~~ 3
~~3~~ ~~8~~ ~~1~~
~~2~~ ~~7~~ ~~5~~ 6 (2 6 4
~~2~~ ~~9~~ ~~9~~ 9
~~2~~ ~~2~~

And then do I say forth, 4 times 9 maketh 3 6, which I take from 3 6, & there remaineth nothing, so that the quotiente of thys diuisiõ, where 7 6 5 6 is diuided by 2 9, is 2 6 4, which doth declare, that if 2 6 4 be multiplied by 2 9, ÿ summe wil bee 7 6 5 6. And thus I perceiue now how bothe Multiplication is proued by Diuision, and Diuision also by Multiplication.

M. Now haue I ended the fyue most common kindes of Arithmetike, for as touchinge

Mediation, Duplation, Triplation, and suche other, they are no seuerall kindes of Arithmetike, but ar cōtained vnder the other: for Mediation is contained vnder Diuision, and is nothing else but diuiding by 2: and so are Duplation and Triplation contained vnder Multiplication: for Duplation is nothinge else but multiplying by 2, and Triplation is multiplying by 3, of which I wil onlye propose example s, for the rules you haue hearde already.

An example of Mediation.

If you wold Mediate or diuide into 2 this sum, 4 5 3 1 0 1 0, you shall set 2 for ẏ diuisor, & work as you learned before, as thus.

4 5 3 1 0 1 0 (
2

Then I fynde 2 in 4 two tymes, therfore my quotiēt must bee 2, so I cācell 4 & 2, and remoue the diuisor forwarde thus.

~~4~~ 5 3 1 0 1 0 (2
~~2~~ 2

Thē againe I finde 2 in 5 twise, & 1 remaining, so I write 2 againe for my seconde nūbre of ẏ quotiēt, & cācell 5 & 2, & ouer 5 I set 1, thus.

1
~~4~~ ~~5~~ 3 1 0 1 0 (2 2
~~2~~ ~~2~~

Then remoue I the diuisor forwarde and seeke a newe quotient, which is 6; then say I, 6 times 2 make 1 2, take ẏ out of 1 3, & ther resteth 1, so I cācel 2 & 1 3, & ouer 3 I set, 1 thus

~~1~~ 1
~~4~~ ~~5~~ ~~3~~ 1 0 1 0 (2 2
~~2~~ ~~2~~ ~~2~~

Th

Then remoue I the diuisor forward, and seeke a newe quotient which is 5 ; then take I twise 5 out of 1 1, and ther resteth 1 ,so I cancel the 2 and the laste figure of 1 ,& let the first stande, thus.

~~1~~ ~~1~~
~~4~~ ~~5~~ ~~3~~ 1 0 1 0 (2 2 6 5
~~2~~ ~~2~~ ~~2~~ ~~2~~

Then remoue I the diuisor forward and seke a new quotient, which is 5 : thē take I 2 fiue times out of 1 0, & ther resteth nothing.

~~1~~ ~~1~~
~~4~~ ~~5~~ ~~3~~ ~~1~~ ~~0~~ 1 0 (2 2 6 5 5
~~2~~ ~~2~~ ~~2~~ ~~2~~ ~~2~~

Then remoue I againe the diuisor forwarde, thus.

But bicause I can not find ỷ diuisor in ỷ numbre ouer it, I must set a cyphre in ỷ quotient, and remoue the diuisor to the next place, as appereth in the figure before.

~~1~~ ~~1~~
~~4~~ ~~5~~ ~~3~~ ~~1~~ ~~0~~ 1 0 (2 2 6 5 5 0
~~2~~ ~~2~~ ~~2~~ ~~2~~ ~~2~~ ~~2~~ ~~2~~

Then seke I a new quotient which I find to be 5 ,for so many times may I haue 2 in 1 0. Then haue I fullie ended this Mediation or diuision by 2 ,ond the quotient is this, 2 2 6 5 5 0 5 ,which is ỷ halfe of 4 5 3 1 0 1 0 as you may trie by Duplacion: for double that quotient, or multiplie it by 2 , and the same number will amount. Duplation

I will no longer tarry about these, seyng they are but membres of the other kindes.

Easy formes of multiplication.

But here nowe will I teache you certaine easie formes both of Multiplication and of Diuision, and firste of Multiplication.

If you would therfore multiplie any sum by 1 0, you shall nede to do no more but adde a cyphre before his firste place: as for example: 3 6 multiplied by 1 0, make 3 6 0. Likewaies if you will multiplie any summe by 1 0 0, put two ciphers at his beginning.

So if you woulde multiplie any summe by a thousand, adde three cyphers to the beginning of it.

Scholer. This doo I well perceiue: and also the reason of it.

Ma. I will omitte all reasons til our next meting, whē I shal tell you ÿ reason of al other partes of Arithmetike also: & as to our matter now, loke (as I haue tolde you) ÿ you both remembre it, and also often practise it.

But if you would multiplie any number by 5, marke first whether the numbre be euē or odde: and if it be euen, take the halfe of it, and write a cyphre at the beginning of it, as for example: I wold multiplie 2 5 6 4 by 5, I take the halfe of it, whiche is 1 2 8 2, (as you may knowe by Mediation) and before it I set a cyphre thus, 1 2 8 2 0, and this is 5 times 2 5 6 4.

And thus maye you doe with any other euen summe that you would multiply by 5.

But if the summe be odde, as for exampl

2 5 6 3

2 5 6 3, then must you take the lesser halfe of it, or (if you wil) take away 1 from the first figure (as here take 1 from 3) and then take the halfe of the reste, and set before it 5: as of 2 5 6 3, the lesser halfe is 1 2 8 1, for here I take but 1 for ẏ halfe of 3, and if I put 5 before that lesser halfe, then haue I multiplied it 5 times, as thus, 1 2 8 1 5.

Sc. What meane you by that lesser halfe?

Ma. There is no iuste halfe of any odde number, therfore if we diuide an odde number into two partes as nighe equall as canne be, yet will the one halfe excede the other halfe by one, as for example. The two moost nearest halfes of 9, are 5 and 4, and likewise of 1 5, are 7 and 8, wher you se that the one parte still is greater then the other by one. Nowe it is easie to knowe which is the greater halfe, and which is the lesser halfe.

Sc. Then I perceiue you, and can do like waies (I doubt not) with any summe. For if it be not very easie to parte into halfe, then will I do it by Mediation easelie inough.

M. That is a sure waie. And nowe haue you learned howe to multiplie easilie by 5, 50, 1 0 0, 1 0 0 0: and of like maner may you do with any other of that sorte.

But nowe if you wil multiplie by 2 0, 3 0, 4 0, & so forth: or by 2 0 0. 3 0 0, & such like, where there is one cyphre in the firste place, or many orderlie in the fyrste places,

 you

you shal take away those cyphres, and mul-
tiplie the sum onely by the other figure, or fi-
gures (if they be many) and then at the begin-
ning of the sum that amounteth, shall you set
so many cyphers as you toke awaie.

Example of 1 8 7 3, which I wold multi-
plie by 3 0 0. First I caste away the two cy-
phres frõ the multiplier, & I multiply ẏ sum
by onely 3 that is lefte, and it amounteth to
8 6 1 9: before which I put the two ciphers
ẏ I toke away before, & then is it 8 6 1 9 0 0,
And that is the summe that amounteth, when
2 8 7 3 is multiplied by 3 0 0.

S. And if ther wer two or more figures besid
the cyphers, I must only take away ẏ ciphers
and multiplie by the other figures, as I lear-
ned before, as if I wold multiplie 9 3 6 4 8
by 2 5 0 0 0, I shoulde take away the thre
cyphers, and multiplie the same by 2 5, an
then at the beginning of that totall sum shul
I adde the three cyphres againe.

M. Euen so, but and if it chaunce the num
bre that shoulde be multiplied, or bothe th
summes, as well the number that shoulde b
multiplied as the multiplier, to haue cyphre
in their firste places, euermore caste awa
the cyphres, and worke by the reste: But r
member to restore as many cyphres to t
amounting summe, as you bated before, as
this example: 3 0 2 0 0 shalbe multiplied
2 0 6, I shal only take away the two ciphe
fro

from the greater numbre, and then multiplye 302 by 206, & afterward adde ŷ two cyphres againe. But if I woulde multiplie the same 30200 by 2060, I shal not only take away ŷ two ciphres from the nūbre that shoulde bee multiplied, but also I maye take awaye the one cyphre from the multiplier, & then muste I adde 3 ciphres to the sum that amounteth: but take heede that you take awaye no cyphre that commeth after anye signifying figure, as in this last example you may not take away that in the fourthe place of the higher numbre, nother that in the third place of the multiplier: how be it, yet this you may doo: If one cyphre or more come in the middest of your summes, you may multiply by the other figures, and ouerskippe them, but so that you gyue euery figure his due place, as thus: I will multiplye 3026 by 2004, therfore I set theim thus.

3026
2004
24
1208

And thus I multiplye them: first 4 times 6, make 24. I set the 4 vnder the firste place, and keepe the 2 in my mynde, or wryte it downe for easye remembraunce: than say I againe: 4 times 2 maketh 8, and 4 times 0 maketh 0, then 4 tymes 3 maketh 12.

But now whan I come to the next cyphre bicause that it multiplieth nothing, I let it go

go, and lyke wayes the seconde cyphre, but then whan I do come to the 2, and multiplye it into the 6 of the ouer numbre, you muste take heede (accordinge as I taught you in multiplication) that the fyrste numbre amountinge of the Multiplication, bee set vnder the multiplier righte, and the other orderly towarde the lefte hand, according as you maye see in this example.

3	0	2	6
2	0	0	4
1	2	0	8
	1	2	
6	0	4	

Where yf you had expressed the ciphres after the common rate, than shoulde the figures stande as foloweth.

But in effect all is one, saue that the first waye by ouerskipping of the cyphres is the shorter and easier waye: for that in effect they be bothe one, the addition of the parcels wyll declare, whiche in both wyll appeare thus.

		3	0	2	6
		2	0	0	4
				2	4
		1	2	0	8
		0	0	0	0
		0	0	0	0
			1	2	
6	0	4			
6	0	6	4	1	0

And nowe wyll I make an ende of this matter.

Sch. Syr I thank you, for I se great ease in this waies of Multiplication. and if you canne shewe me suche lyke in Diuision, you shall greatlye further me.

Mast. Yes I will teache you some eas[ie] wayes in Diuision also, and first this: If y[ou] wolde diuide any summe by 1 0, you shal o[n]ly wi[...]

ly wyth your penne make a square lyne bee-
tweene the fyrste fygure of your summe and
the second, and than haue you doone: for the
whole numbre that foloweth the lyne stan-
deth for the quotiente, and the fygure that
is before the line, is the remainer: as for ex-
ample 3 6 4 8 deuided by 1 0, wil stād thus
where 3 6 4 is ỷ quotiēt, & beto-
keneth ỷ so many times are 1 0 in 3 6 4 8: & the 8 after ỷ line is the remay-
ner which cā not be deuided into 1 0, but by
breakinge it into fractions, wherwith I wil
not meddle yet.

364 | 8

Easy formes of diuision.

And so like waies, yf you woulde deuide
anye summe by 1 0 0 wyth youre pen, you
shall cut away the twoo firste figures: and yf
you woulde diuide by 1 0 0 0, you must cut
away the 3 first figures: and so of anye other
diuisor, whose laste figure is 1 and the other
be cyphres, loke howe many ciphres the dy-
uisoure hathe, and so many fygures at the
beginninge shal you cut away with the squire
lyne, & they stande alwayes for the remainer,
bycause they are lesse then the diuisour, and
canne not be diuided by it: and the other fy-
gures that be behinde the line, stande for the
Quotient.

But nowe yf your diuisoure haue anie o-
ther fygure in hys laste place than 1, and in
all his other places haue cyphres, loke howe
many cyphres there be, cutte awaye so many
of the

of the first figures of the numbre that shoulde bee diuided , and diuide the reste that foloweth the lyne, by that fygure that is in the laste place , as if it were the whole diuisour.

Example of 6 4 2 8 4, which I would diuide by 3 0 0, here must I cut away the two fyrste fygures, (for so many ciphres my dyuisour hath) and muste deuide the reste by 3, whiche is the fygure in the laste place of the diuisour. Fyrste therefore I parte awaye the two fyrste figures, & the sum standeth thus.

6 4 2 | 8 4.

Than do I diuide 6 4 2 by 3, and the quotient is 2 1 4, for in 6 I fynde twise 3, and in 4 ones, and 1 remaininge, whiche 1 wyth the 2 nexte before doothe make 1 2, wherein I fynde 3 foure times. and thys is a ready waye to tourne shyllinges into poundes: for syth 1 pounde dooth containe 2 0 shillinges, I muste dyuide the whole numbre of shillinges by 2 0, therefore easily to do it, I see that my diuisour hathe one cyphre, and therefore I cute awaye one fygure from the begynninge of the whole summe of shillinges and than do I mediate or diuide by 2 the other figures or summe that foloweth.

Scholer. I wyll put an example.

If you wold diuide 6 4 2 8 7 shillings by 2 0, ye is to say: If I wolde tourne so many shillings into poūds, I must cut away ye first

figu

figure, that is 7, & diuide the rest, ẏ is 6 4 2 8 by 2, so shall the Quotiente be 3 2 1 4, wherby I know ẏ 6 4 2 8 7 shillings make 3 2 1 4 pound, and 7 shillinges remaining.

Mast. Nowe proue by Multiplication whether you haue well done or no.

Sch. The quotient is 3 2 1 4, which I do multiplie by the diuisoure 2, and it doeth amount to 6 4 2 8.

M. Hereby may you perceiue not only that you haue well done, but also howe by diuision you may turne shillings easilie into poundes: And contrarie waies, by Multiplication you may tourne poundes into shillinges.

But here shall you see amongest diuers men, diuers formes of suche diuision, but yf you marke what I haue tolde you, you shall perceaue easilie all their waies: for some men did not cut awaye so manye of the firste figures of the summe that they woulde deuide, as ther are cyphers in the first places of their diuisor, but they set all their cyphres orderly vnder the firste places of the numbre that they would diuide, and than with the other figure (or figures if they be many) they diuide the reste of their summe.

Example. If they would diuide 7 2 5 9 3 1 by 3 4 0 0 they set their summes, thus.

7	2	5	9	3	1
3	4			0	0

And then do they diuide orderlie, till they come to the cyphres; for there they staye and

ende

ende their worke, as in this example: They seeke how often 3 may be found in 7, which is 2 times, and 1 remaining, therfore they sette 2 in the quotient, & cancell 3 and 7, & ouer 7 thei set that 1 that remaineth, thus.

1
~~7~~ 2 5 9 3 1 (2
~~3~~ 4 0 0

~~1~~ 4
~~7~~ ~~2~~ 5 9 3 1 (2
~~3~~ ~~4~~ 0 0

Then do thei go forthe saying: two times 4, maketh 8, which they take out of 1 2, & ther remaineth 4, thus.

Then renewe they the diuisor forwarde, and seke howe often 3 may be founde in 4, which is but ones, and 1 remaineth, thē set they 1 in the quotient, and cancel 3 & 4, and ouer thē they set that 1, thus.

1
~~1~~ 4
~~7~~ ~~2~~ 5 9 3 1 (21
~~3~~ ~~4~~ 4 0 0

Then take they ones 4 oute of 1 5, and ther resteth 1 1: or els more easily: Take ons 4 out of 5 and there resteth 1, so thei cance the 4 and 5, and sette one ouer them, thus

1
~~1~~ ~~4~~ 1
~~7~~ ~~2~~ ~~5~~ 9 3 1 (21
~~3~~ ~~4~~ ~~4~~ 0 0
~~3~~

Then sette they forth the diuisour againe, and seeke how many times 3 are in 1 1, which thei finde 3 times, & 2 remaining: so theý set 3 in ẏ quotiēt, &

~~1~~ 2
~~1~~ ~~7~~ ~~4~~
~~7~~ ~~2~~ ~~5~~ 9 3 1 (31
~~3~~ ~~4~~ ~~4~~ 4 0 0
~~3~~ ~~3~~

cance

canceth 1 1 & 3, and ouer the setteth 2, thus.

Then doe they multipli 4 by 3, which maketh 1 2, y withdrawe they out of 2 9, & there resteth 1 7, of whiche y 7 muste be set ouer the 9, and the 1 ouer, 2, thus

```
  1
 x2
x4x7
72593 1 ( 2 1 3
34400
 33
```

And now are y two cyppres next ensuing so y the diuisor can no more be set forwarde, and therfore is the diuision ended, and the remainer is 1 7 3 1.

Nowe the quotient, which is 2 1 3, dothe declare, y if you diuide 7 2 5 9 3 1 by 3 4 0 0, you shall fynde it ther in 2 1 3 times, & there remaineth 1 7 3 1, so shall you fynd it, if you worke as I taught you by cuttinge away y two first figures, bicause of y two cyphres. But this muste you marke (as you may perceaue by this laste exãple) y if there be left any other remainer in the sum y was behind the squire line, y the remainer must be set to the latter ende of the firste remainer, whiche was cut away with the squire line: as you woulde diuide 7 2 5 9 3 1 by 3 4 0 0 after the forme y I taught you, thã wold your summes appeare thus.

```
    1
   x2
  x4x7
  7259|31 ( 2 1 3
  344.
  33
```

So that 17, whiche remaineth after the line, muste be sette to the 31 (that was cutte away with the line) in higher places, as you see here: where that 17 with the 31 do make 1731.

Another inuention of easye Diuision

And here would I make an ende of Diuision, sauinge that there commeth to my mynde one late inuention of easy Diuisyon, which I wyl briefly let forth to you, so that yf you fynde ease in it, you maye vse it. Bycause that the hardest point in Diuision is the ready and easy fyndyng of the quotient nombre: and againe, yf that bee truely knowne, all the reste is but lyght to bee done: therefor this waies shall you quickly and truely fynde, the Quotient.

Fyrst write the nine fygures of nūbre: I meane 1 2 3 4 5 6 7 8 9, not along as I haue set them now, but vp & downe, as in this forme. And at the lefte syde of them draw a long lyne as you see here: then considre the diuisoure by which you intend to worke, and sette it on the lefte syde of the longe lyne, right against 1, and for a distinctiō drawe a line vnder it: then multiply your diuisor ordrely by eche of those fygures, beginninge with 1, and so go downewarde tyll you haue ended all: And looke what doo[th] amount of the multiplication of eche fig[ure]

1
2
3
4
5
6
7
8
9

into the diuisor, than do write againste the fi-
gure where by you did multiply.

Scholar. By example I may perceiue it better.

M. Take this exãple, 263845 to be deuided by 64, then muste I sette the 9 figures as I sayde before, and the diuisour muste I sette against the 1, thus.

64	1
	2
	3
	4
	5
	6
	7
	8
	9

Than must I multiply that diuisour by eche figure ordrely: firste by 2, & it maketh 128, which I must set againste 2 at the left hãd.

Than multiplye 64 by 3, and it maketh 192, which is set againste 3. Then 4 times 64, make 256, that set I by 4. Then say I 5 times 64 make 320, that sette I agaynst 5. Than 6 times 64, make 384, that set I against 6. Thẽ 7 times 64, make 448, whiche I set against 7.

Further I say: 8 tymes 64, make 512, which I sette by 8. And laste of all I saye: 9 times 64, make 576, which I set against 9. And than thei will stande thus.

64	1
120	2
192	3
256	4
320	5
384	6
448	7
512	8
576	9

And so is the table ended, by whiche you may easylye fynde the Quotient, as you shall see by example nowe.

Doe you set downe the numbres (as you

learned before) according to the ordre of Diuision. 263845

Scholar. That is thus. 64

M. Now loke what nūbre standeth ouer the diuisor, reckninge therto all theim that bee behinde yt towarde the lefte hande.

S. Then are there ouer the diuisour 263.

M. That is iuste: nowe seeke in the table on the left syde, whether you can finde 263.

Scholar. It is not there.

M. Then take that numbre that is nexte to it, beneth it: I meane a lesser numbre then 263, but of al the lesser numbres ỷ the table hath, take you that, ỷ goeth nighest to 263.

Scholar. That is 256.

Ma. So is it: and marke this euermore, whan you can not finde iustelye in the table that summe that is ouer your diuisour, then note that that is nexte beneth it of anye sum that is in the table, and looke at the right hā of the lyne what figure or digitte that is against that summe, and take that digite fo your quotient, & then worke on, as you lear ned before: for now haue I tolde you ỷ whol vse of this table.

Howbeit, yet that you may bee sure understande it, I will see you ende this e sample of Diuision by it.

Now therefore begin againe. 26384

S. First I set down ỷ sūmes, 64

after the common manner, thus.

Then doe I looke ouer the diuisor, and finde there 263. Nowe to know how many times 64 may be taken out of 263, I resort to the table aforesaide, and seeke for the numbre 263, but it is not there, therfore as you badde mee, I take a lesser numbre, the next to it that I can fynde in the table, and that is 256, whiche numbre hath against it on the righte hande this digitte 4, whiche I muste take for the fyrste fygure of my quotient.

Then doe I (as I learned before) multiply that quotient into euery figure of the diuisor ordrlye, withdrawinge the summe thereof, amounting out of the ouer summe. as here I say first: 4 times 6 make 24: so I take that out of 26, sayinge: 4 out of 6, remaineth 2, which I write ouer the 6: then 2 out of 2 remaineth nothing. then cãcell I 2 & 6, & also 6 in the diuisor, & the sum standeth thus.

$$\begin{array}{l} \phantom{\cancel{2}}2 \\ \cancel{2}\cancel{6}3845(4 \\ \cancel{6}4 \end{array}$$

Then doe I lykewise saye forthe: 4 times 4 make 16, which I take out 23, and there resteth 7 to be sette ouer 3, so that 3 with the 2 behinde it: and the 4 vnder it, muste bee cancelled, as you see here.

$$\begin{array}{l} \phantom{\cancel{2}}\cancel{2}7 \\ \cancel{2}\cancel{6}\cancel{3}845(4 \\ \cancel{6}\cancel{4} \end{array}$$

Then haue I done wyth the figure of the quotient.

M. Nowe set forwarde your diuisour, and seke a newe quotient, as you soughte this.

~~2~~7
~~2~~~~6~~~~7~~845 (4
~~6~~~~4~~4
6

Scho. Than thus standeth the fygures, so that ouer the diuisor I se 78, which I seke in the table, and can not fynde it: therfore I take the next lesser, and ỷ is 64, the diuisour it selfe.

M. So muste you do when there is none other.

Sch. Than againste it I fynde this digit 1, whiche I must set in the quotiente before 4, thus.

~~2~~7
~~2~~~~6~~~~7~~845 (41
~~6~~~~4~~4
6

Then multiplye I 6 by 1, and it is but 6 still.

Note. Ma. You neede not go about to multiplye whan the quotient is 1, for 1 dothe nother multiplye nor diuide, but in suche case onelye subtracte the diuisor out of the numbre that is ouer it.

S. Then I take 4 oute of 8, and there resteth 4, and 6 out 7, there remaineth 1, so I cancell those numbres, & write ỷ remainers ouer their places, thus.

1
~~2~~~~7~~4
~~2~~~~6~~~~7~~~~8~~45 (41
~~6~~~~4~~~~4~~
~~6~~

Then set I forwarde the diuisour againe, thus.

Wher

Where I see ouer the diuisour 144, which I seke in the table, & find it not: therfore I take the nūber in the table, þ is nexte therto beneth it, which I finde to be 128, againste whiche in the ryght syde I finde 2, which I take for my quotient, and that doe I multitiplye firste into 6, and therof commeth 12

```
     1
    174
  163845  (41
    6444
     66
```

whiche I take out of, 14, and then remayneth 2. that 2 I sette ouer 4, and cancell the other fygures, 1, 4, & 6, thus.

```
     12
    174
  163845  (412
   6444
    66
```

Than say I forth: 2 times 4 are 8, whych I take oute of 24, and there remayneth 16, of whiche I wryte the 6 ouer 4, and the 1 ouer 2, and cancell 2, 4 and 4, thus.

```
      1
     12
    174
  163845  (412
   6444
    66
```

Now againe I set forwarde the diuisor, & thus.

```
      1
     11
    1746
  163845  (412
   64444
    666
```

and seing ouer it 165 I seeke that in the table, but fynde it not, therfore I take the

next lesser, whiche is 128, agaynst which I fynde 2: y so I set into the quotient, an by it I multipli first 6, and thereof commeth 12, which I take out of 16, and ther resteth 4, the cancel I 1, 6, and 6, & ouer 6 I set 4, thus.

```
        1
       1 2 4
      2 7 4 6
   2 6 3 8 4 5 (4 1 2 2
    6 4 4 4 4
     6 6 6
```

Then doe I multiplye 4 by 2, and it maketh 8: whiche I take out of 45, and there remaineth 37, as in thys exaumple.

```
         1 3
        2 2 4
       2 7 4 6 7
   2 6 3 8 4 5 (4 1 2 2
    6 4 4 4 4
     6 6 6
```

And nowe haue I doone.

M. Wel, nowe I see that you can worke by this kinde of diuision, as farre forthe as I taught you.

S. Yea sir, I thanke you, and I fynde in it muche ease and certainesse.

M. Yet one thinge I doubt whether you perceaue: What if you did fynd in the tabl the number that standeth ouer the diuisour what would you next doe?

S. I thinke I should take the digit again it on the lefte hande for the quotient.

M. So is it: and as often as you seke the table and find your numbre iuste, the di gite against it is your true and iust quotient

I call that a true quotient, if it bee the right quotient that you should take, though your diuisor multiplied by the same, doe not clere ly subtract ÿ numbre ouer it, but ther dothe sumwhat remaine, as it chaunced in all the examples that you dyd worke by. But yf it should chaunce, (as it dooth often) that youre diuisoure multiplied by your quotiente, doo subtracte cleane the numbre ouer it, then call I that quotient not onely a true quotiente, but also a iuste quotient, because it doth iustlie consume the numbre ouer the diuisor: and that chaunceth euermore whan the numbre ouer the diuisor is iustely founde in the table.

Scholer. This I shall remembre.

Master. But yet one easye poynte more I will tell you in this sorte of diuision, therefore marke it well.

Whan you haue found in the table, other the same summe that is ouer the diuisour, other the nexte beneth (for lacke of the other) than looke what digite standeth againste it, take that for your quotiente: And because it is some payne to multiplie the diuisour by the quotient, you shall not need to do it, but only take the numbre that you found in the table, and subtract that from the ouer numbre: for if you doe multiply the diuisour by the quotient, that wyll be the nūbre that shal amoūt. Therfore is thys way more easyer.

S. So is it, & also more certainer for such

 as

as I ame, that mighte quickly erre in multi-
plyinge, especially beinge smally practysed
therein.

Ma. Then proue in some briefe exãple whe-
ther you cã do it, and so wil we make an ende.

S. I woulde diuide 38468 by 24, ther-
fore firste I set the table as here foloweth.

24 | 1 Then set I the two 3846[8]
48 | 2 ssies of Diuision thus. 24
72 | 3 And ouer the dyuisour I finde 38,
96 | 4 which I seke in the table & fynde
120 | 5 it not, therfore take I the next be-
144 | 6 neth it whiche the table hath, and
168 | 7 that is 24, ẏ diuisor it selfe, against
192 | 8 whiche is set 1, whiche I take for
216 | 9 the quotiente, whiche I set in hys

place. and now I nede not to multiplye the
diuisor by yt, but onely to
withdrawe the deuisor out 14
of the 38, that is ouer it, & ~~38~~468 (1
so remaineth 14, as thus. ~~24~~

Then set I forward the
diuisoure, and finde ouer

14 144 as appeareth: the se[ke]
~~38~~468 (1 I that numbre in the tab[le]
~~24~~4 and finde it, and against [it]
12 is 6, therfore I sete 6 be-
fore 1 for my quotient, a[nd]

I take that 144, for the iust multyplicati[on]
of the diuisoure by that quotient. and the[r-]
fore without any newe multiplication I [do]
subtra[ct]

[S]ubtract that 144, from the other 144, and ther resteth no thinge, as you may see.

14
38468(16
244
2

Therfore I set forwarde the diuisour, but seeynge it wil not be in ẏ nexte place, (for then ouer 2 woulde be nothynge) I set it forwarde twise, as you see here.

14
38468(16
244
22

And for bycause that I coulde not sette it in the next place folowing, therfore I sette a cyphre in the quotiente, as you see.

Than looke I ouer the diuisor, and finde 68, which I can not finde in the table, therfore take I the next beneth it, which I fynde in the table, and that is 48, and agaynste yt standeth 2, which I take for the quotiente. And then without any multiplying of the quotient in to the diuisour, I doo subtract that 48 from 68, & there resteth 20, as heere appeareth.

14 20
38468(1602
24424
22

And so haue I ended the whole diuision.

M. Now can you sufficietly skill in these kindes of Arithmetike. And nowe for the vse of these two laste, that is Multiplication and Diuision, I will brieflye shewe you the feate of Reduction.

REduction is, by whic
al summes of grosse de
nominatiõ may be tou
ned into sũmes of mor
subtill denomination
And cõtrary waies: al
sũmes of subtyle deno
minatiõ may be brough
to sũmes of grosser de
nomination.

Scholer. What call you Grosse denomi
nation, and Subtile Denomination?

Grosse de nominatiõ

M. That I call a Grosse denomination, whi
che dothe containe vnder it manye other sub
tiler, or smaler: As a pounde in respecte
shyllinges, is a grosse denomination, for
is a greater then shyllinge, and containe
manie of them: And shillinges in compar
son to poundes, are a Subtile denominati
on, for bicause they are lesser then pounde
and manye of them are contayned in one
the other: as so lykewaies of other thing
what so euer thynge is compared to othe
if it be a greater and containeth many of th
yt is a grosser denomination: but if yt b
lesser, so that many of them are in the othe
then are thei called Subtile denominatio
whereby you may perceiue, that one den
mination may bee called a grosse denomi
tion, and also a Subtyll (that is to saye

Subtile denomi- nation.

gre

greate and small) in dyuers comparisons. For shillinges compared to poundes, are a subtyle or smale denomination: but cōpared to pennies, they are a grosse or great denomination.

Scholer. Nowe I vnderstande the name, I praie you teache me the vse.

M. The vse is easilye learned, yf you remember what you haue learned before. For yf you wyll reduce any summe of a Grosse denomination into a summe of a Smaller or subtyler denomination, you muste consydre howe manye of that Subtyler denomination doo make one of the Grosser denomination, and by that numbre or numerator dooe you multiplie the other summe: as if you woulde reduce 20 poundes into shillinges, you muste considre that in a pound are included 20 shillinges, therfore multiplie the one 20 by the other 20, & there will amount 400, whereby you may knowe, that in 20 pounde ar contained 400 shillings. Likewaies if you wolde reduce 30 shyllinges into pennies, consyderinge that in 1 shilling are 12 pennies, you must multiplie 30 by 12, & it will be 360. Whereby you finde, that in 30 shillinges are contayned 360 pennies. And thus may you reduce any grosse denomination into a more subtiler, by multiplication, if you know how manye of the lesser doo make the greater: of whiche thing I will anone giue you a briefe table for the moste accustomed kyndes of mo-

ney, weightes, measures, and tyme, and suche
lyke, whereby you maye knowe howe often
eche subtile denomination is contained in the
grosser, whan you shall neede it for the fore-
sayde kinde of Reduction. And also the same
shall serue you, if you woulde reduce anye sum
of a subtiler denomination, into a summe of
a grosser denomination: For in suche reduc-
tion you must considre (as in the other forme)
how manye of the smaller do make the grea-
ter, and by that numbre muste you diuide th
other summe, and the quotiente wyll declar
howe manye of the greater denomination ar
comprehended in that summe: as for exam
ple: If you woulde knowe howe manye shy
linges are contained in 3240 pens, consid
that 12 pennies do make 1 shilling, you mu
diuide that 3240 by 12, & your quotiēt w
be 270, wherby you know that so many shy
linges are in 3240 pennies. But & you wo
knowe farther, howe manye poundes are
those 270 shyllinges, seeing that euery pou
conteineth 20 shyllinges, dyuide that 27
by 20, and it will be 13, and 10 remainin
wherby you may knowe, that in 3240 pe
nies, or 270 shyllinges, are 13 poundes a
10 shillinges. For euermore the remayn
must be named by the name or denominati
of the sum that was diuided, whiche in th
place were shillinges. And thus may you
with any other kindes of denominations.

Wheres

Wherefore to the intente that you maye haue a light knowledge in the cōmon coines, weightes, measures, and suche other, I haue prepared here a briefe table, whiche shall suffyse to you at this time, till hereafter at more conuenient opportunitie I may instructe you more exactly in the same.

A table for English coynes.

English coines.

A Soueraygne.	A quarter Noble.
Halfe a Soueraygne.	A Croune.
A Royall.	Halfe a Croune.
Halfe a Royall.	A Grote.
A quarter Royall.	A harpe Grote.
An olde Noble.	A penny of 2 pence.
Halfe an olde Noble.	A dandy pratte.
An Angell.	A penny.
Halfe an Angell.	An halfe penny.
A George Noble.	A Farthinge.
Halfe a George noble.	

The valewe of Englishe coynes.

The valewe of Englishe coines.

A Soueraine is the greatest englishe coine, and cotaineth 2 Royalles, or 3 Angelles, eyther 9 halfe crounes, or 4 croumes and an halfe, that is to say, 22s 6d.
Halfe a Soueraigne is equal with a Roial.
A Royall containeth an Angell and a halfe, that is to say. 11s. 3d.
Halfe a Royall contayneth 5s, 7d, ob.
A quarter of a Royall contayneth 2 shyllinges,

ges, 9 d. ob. q.

An olde Noble, called an Henrie, is worthe 2 crounes, or a noble and a half, that is 10 s.

Halfe an olde Noble is worth 5 s.

An Angell contaimeth a croune and halfe or 3 halfe crounes, that is 7 s, 6 d.

Halfe an Angell is worth 3 s, 9 d.

A Noble called a George, is worthe 6 s, 8 d.

Halfe a Noble is worth 3 s, 4 d.

A quarter of a Noble (which in the olde statutes is caled a farthinge) contayneth 20 d.

A croune contayneth 5 s, & the halfe crowne 2 s, 6 d. Howe be it there is an other crowne of 4 s, 6 d. whiche is knowen by ye rose side: for ye rose hathe no croune ouer it, as in ye other crowne, but it is enuyroned on ye 4 quarters with 4 floure delucis, wherby you may best knowe it. But I will returne to speake of the valeue of the coynnes, for I intende not now to describe the formes of them. Now of golde are there no more common coynes.

In syluer, the greatest is a Grote, which contayneth 4 pennies. Than is there another Grote called an Harp, which goeth for 3 d. Then nexte is a Pennye of 2 d. And then Dandye pratte, worth 3 halfe pence. Nexte a Penny, then halfe a peny, and laste & least of all a farthinge, whose coyne ys on the one side a crosse, and on the other syde a perc... les. Thys I tell you, bicause I see many that can not knowe a farthinge from a sm...

hal-

pennye.

Nowe haue I tolde you all the englishe coynes bothe of golde and syluer, but yet of the two moste common valowers of money spake I nothinge, that is to say, of poundes and shillinges, whiche though they haue no coynes, yet is there no name more in vse then they: of which the shillinges containeth 12 pennies or 3 grotes, and the pounde 2 old Nobles, 3 George nobles, or 4 Crownes, that is to say, 20 s.

Here would I now expresse the valewes of sundry other coynes of dyuers countries, but for three causes I nowe refrayne. The firste and chiefeste is, bicause they are not currant by the statutes of this realme. An other cause is, by reason they are so vncertaine, that they bee neuer longe at one rate. And agayne they are so different in so manye places, that it were matter inough for a greate booke to speake sufficientlye of them all. Howbeit, yet bycause you shall not bee alltogether ignorante of them, I will shewe you the valewes of some that are most in vse: and firste of Fraunce.

The moste common money are Deniers, Soulx and Frankes. 12 Denyers make 1 soulx, 20 solx make 1 Franck: so that as you see these 3 kindes are like in the rate to pennies, shillinges and poundes with vs, but this is the difference, that their Deniere Frenche coynes.

is but the 9 parte of our pennie, and so theire soulx (commonlye called sowles) go 9 to oure shillinge, and 9 of theire Francs to an englishe pounde of money: so that 3 of theire Francs make a noble. And by those 3 may you practise how to reduce Frenche monye into Englishe mony. And as for the rest of theire coynes I will omit tyll an other time, when I entende to shew you the rate of sundrye other kindes of money.

Flaundres coynes.

But nowe as for the coynes of Flaundres be so changeable, that you muste knowe them from time to time, else you can not reduce them into our mony, certainlye. But yet bycause that you shall haue an example of theire money to exercise you withall, you shall take those that bee moste common, as Styuers, bothe syngle and double, Grotes flemmishe, Carolus, and Gyldens: A flemmishe Grote is a lyttle aboue 3 farthinges Englishe. A single Styuer is 1 d, ob. q. The double Styuer is 3 d, q. The siluer Carolus single, is 2 d, q, q, c. The double siluer Carolus, is 4 d, q, q. Then is there also the Carolus gylden, whiche is worth 20 styuers. And the Flemmyshe noble is worth 3 Carolus gildens, and xij styuers.

But I wyl let them passe now, exhorting you to practise to reduce those kyndes in Englishe money, accordinge as I haue set forth here folowinge: 2160 deniers, ma

240 d, or 20 s. 3240 deniers, make 360 d or 30 s: 8352 deniers, make 928 d, or 3 li. 17 s 4 d: 2160 soulx, make 240 shillings: and so of other in like rate.

But if you will reduce Flemmishe monye iustly, you must reduce it firste into the smallest part of english money that is in the coyn, as for example: If I wold reduce 368 double styuers into Englishe money, consideringe that a double Styuer containeth 3 d, q̄, you shall firste looke how many q̄, be in the double Styuer, and you shal find them 13, therfore multiplye the summe of the styuers by 13, and than haue you their valewe in farthinges, which is 4784. Nowe if you diuide that by 4, then will there appeare the number of pence: but better it were to dyuide it by 48 (for so manye farthinges are in 1 shilling) and then will the quotient declare the summe of the shillinges.

Likewaies if you woulde reduce anye summe of single styuer into Englishe monye, you must multiplye the summ fyrste by 13, and than haue you the summe of q̄, whyche sum if you diuide by 8, then will amounte the summe of pennies: or if you dyuide it by 96, the summe of shillings will appeare. But this marke in all diuisiō: whan yee reduce to bringe 1 denomination into an other, if there be any remainer after the diuision, that muste be named by the denomi-

 nation

nation of the grosse summe that was diuided, as for example: I wolde bringe 254 q into pence, therfore I doe diuide that 254 by 4, for so manye farthinges make 1 pennye, and the quotiente is 63, which is the summe of the pence: and then remaineth yet 2, whiche are farthinges styll, as one may proue by diuiding. And this muste bee marked in all diuision, namelye when it is done for reduction.

weightes

Thus much haue I sayd of Monye, nowe will I shewe you in like sorte the distinction of waightes, after the statutes of Englande, where the leaste portion of waighte is commonlye a Grayne, meaninge a grayne of corne or wheate, drye, and gathered out of the middle of the eare. Of these graynes in time passed, 32 waied iust 1 penny of Troy, and than was but 20 pennies in an Ounce. But now are there 46 pennies in an Ounce, so that ther are not fully 14 graynes in one pennye.

A graine

A penny of troye.

An ounce

But nowe of Ounces after Troye rate (which is the standarde of Englande) 12 O make 1 pounde.

But commonly there is vsed an other waight called Haberdepoise, in whiche 16 Ounces make a pounde. Therfore whan you would reduce ounces into poundes, you muste consider whether youre waightes bee Troy waightes or Haberdepoise: and if it be Troy waygh

Haberdepoise weights.

waight, you must diuide your ounces by 12 to bringe them to poundes, but if yt be Haberdepoyse, you must diuide them by 16. Now agayne, there be greater waights which are called an Hundred, halfe a hundred, & a quarterne, and also halfe a quarterne. &c. — A hundred waight.

Scho. Why? so there may be reckened 20 pounde, 40 pounde 200 pounde, and such innumerable.

M. All these are numbres of waighte, but thei haue no cōmon waightes made to theire rate, as the other haue. And agayne, these that I dyd name, are not juste in numbre, as they seeme by theyr name, for an hundred is not iust 100, but is 112 pounde. And so the halfe hundred is 56, the quarter 28, and the halfe quarter 14. And this is the common waights vsed in moste thinges that are solde by waight.

Howbeit, there are in some thinges other names: as in wolle, 28 pounde is not called a quarterne, but a Todde: and the 14 pounde is not named halfe quarterne, but a stone, and the 7 pound Halfe a stone. Other names because they differ in many places, & agree in fewe, I let theim passe. — Wolle weightes. Todde. Stone.

But a Sacke of woll by the statutes is lymited to be 26 Stone. — Sacke.

Nowe in chese, though it be solde by the hundred, and by the stone in some places, yet the werye weightes of it are Clones, and — Chese waightes

Weyes, so that a Cloue should contayne 7 pounde, and a Weye 32 Cloues, that is 224 poundes. Howbeit some statute bookes say, that a Cloue shoulde be 6 pounde, & some say also ỷ a Weye dothe containe 36 Cloues, & ỷ is cōmonly used, for the cōmon Wey is of 256 li. ỷ is 36 cloues, reckening 7 li. to the cloue, & ther is 4 li. ouer waighe. Let thys suffice you at this time touching waightes.

Cloue. Weye.

Now of waightes are made othere measures, bothe for grayne and lyquor. For a pounde in waight maketh a Pynte in measure, so that 8 Pounde (or 8 Pyntes) doe make a Gallon: halfe a Gallon is named a Pottell: and halfe a Pottell is called a Quarte, which contayneth twoo Pyntes. Nowe aboue a Gallon the next measure is a Fyrkin: then a Tertian, a Kilderkin or halfe a Barrell, and a Barrel. And by those measures are solde cūmonly Ale, Bere, Wine & Oyle, butter, & Sope, Salmon, Heringes and Eeles.

Measures for liquor. A Pynte. Gallon. Pottell. Quart. Fyrkyn. Tertian. Kilderkyn. Barrel.

But as these be unlyke thynges, so the measure of theyre vessels doe differ: for the measures of Ale are as foloweth

Ale measures.

Of Ale.

the Fyrken	containeth	8	gallōs
ỷ Kilderkē		16	
the Barrel		32	

Of beare

the Fyrken	containeth	9	gall[ons]
ỷ Kylderkē		18	
the Barrell		36	

Sop[e]

Sope measures, both Fyrkin, Kylderkin and Barrell, shoulde be all equall to Ale measures. Moreouer the statutes do the limitte ẏ weight of euery of those thre vessels beinge emptye. Sope measures

A Barrell	To	26	
Half Barrell	weighe	13	poundes.
A Fyrken	emptye	6 $\frac{1}{2}$	

Hearinges also bee solde by the same measures that Ale and Sope be solde by. Hearings

Hearinges also are sold by the tale, 120 to the hundred, tenne thousandee to the last.

Salmon and Eeles haue a greater measure Salmon and eeles

	the butte		84	
Salmon	ẏ barrell	hol-	42	gallōs
& Eeles	half bar.	deth	21	
	the firkē.		10 $\frac{1}{2}$	

How be it, some statutes did limit Eele vessels equal with Hearing vessels.

Now as for wine vessels seldom are smaller then Hogges heddes, whiche are of 63 gallons; euery Hogges hed is two Barrels. Yet there are many other wyne vessels, but of them all see this table, and mark the measures one to an other. Wine measures

	the Rondelet		18 $\frac{1}{2}$	
	the Barrell		31 $\frac{1}{2}$	
Of wine	ẏ hogges hed	hol-	63	Gallons
and oyle	the Tertian	deth	84	
	the Pype		126	
	the Tonne.		252	

Tertians But you shall marke that there bee other kinds of Tertians, for there be Tertiãs (that is to saye) Thirdels of Pypes, of Hogges headdes, and of Barrell s, as well of other thinges as of wyne.

Also of Malceweseyes and Secke, &c. the half tonne is not called a Pype, but rather a

A Butt, Butte.

And thus much haue I thoughte meete to tell you at this time.

Scholar. And is that alwaies true?

Ma. I haue tolde you how it shoulde be, but howe it is I may not say, howe they doe differ daily from their iust measure, the Gagiers can tell you better then I. But I will let this passe nowe, and speake brieflye of the other measures.

And as of weightes there dyd spring the liquide measures (wherof I spake laste) so of the same springeth drye measures, as

Dry measures. Peckes, Bushels, Quarters, and suche lyke, wherby are measured corne and like graines, also salt, lime, coales and other lyke. And this is the ordre and quantitie of them.

A pecke. A Pecke is the measure of two Gallons.

Bushell. A Bushell containeth foure Peckes.

Quarter A Quarter holdeth eight Bushels.

Weye. A Waye contayneth six quarters.

These are the common names and measures, but in diuers places there bee diuers sortes.

The bushell in manye places is 2 bushels

bu

but then is the bushell ther called a Stryke. Stryke.
And in some places halfe a quarter is called a Cornocke. But these diuersitis are to many to tell you brieflye them all. And againe sythe they are againste the lawe and statutes, I counte them vnmeete to be vsed. Cornocke.

But nowe remayneth yet an other kynde of Measure, whereby men meate lengthe & breadth, and those are an Ynche, a Foote, and suche other: whose names and lengthe this table sheweth. Measurs to meate. lengthe & breadthe

3 Graynes of barley, make an Ynche. An ynche
12 Ynches, make a Foote. Foote.
3 Foote make a Yearde. Yearde
3 Foote and 9 Ynches make an Elle. Elle.
5 Yeardes and a halfe make a Perche. Perche
1 Perche in breadth and 40 in lengthth, do make a Rodde of lande, whiche some call a Rodde, some a Yearde lande, and some a Farthendele. Rodde Farthēdel

Farthendels, make halfe an acre of groūd.
Farthendels make an Acre.

Here mought I tell you many things els touchinge measure, and also howe to reduce straunge measures to oure measures, but because it can not well be doone wythout the knowledge of Fractions, whiche as yet you haue not learned, I will let theym passe tyll an other time, when I shall instructe you in Geometrie, wherin I shoulde be enforced els to repeate the same often againe. Acre

The partes of tyme.
A Daye.
An houre.
Wecke.
Moneth.
Yeare

Scholer. But yet ſyr of the partes of time, I praie you, tell me ſumwhat.

M. You knowe that a naturall day hath 24 houres, and euery houre hathe 60 minutes. It needeth not to tell you that 7 daies make a Weeke, & 4 Weekes make a common moneth, & 13 monethes make a yeare, lackinge 1 day and certaine houres and minutes, But of that I ſhall inſtructe you hereafter.

Heere will I make an ende of Reduction for this time, whiche thoughe it be counted no kynde ſeuerall of arithmeticke, yet you ſee it is no leſſe needefull to be knowen, nor eaſier to be done, than of any of the other.

S. Mary ſyr it ſemeth vnto me much harder than any other ſorte, for it requireth the knowledge of ſo many thinges. but now ſyr, whē you ſe time, I am ready to learne forth, for as muche of Reductiō as you haue taught me, I remembre, but & if I doe at any time forgette, I ſhall haue recourſe to the tables, whiche you haue ſet forth for me.

M. So do you, for it will not be remēbred without exerciſe. Now Progreſſiō wil I beginne.

PROGRESSION

Although vntil this daye the moſte part of writers haue defined progreſſion as a Compendious kinde of Addition, yet truly it is not ſo, for progreſſion (as the very natur

of the word doth informe any mã, is a going forwarde and proceding in numbres, and that regularly and ordrely, whose place is aptlie chosen to be very nere, or rather nexte after the exposition of the foure principall partes of Arithmeticke, for in it after a moste easye manner, ar al the foure former partes exercised and practised: and not onely Addition, as custumably is done. Whiche custome hathe byn the cause, why it hathe so specially byn named a kinde of Addition, & defined to be a quicke & briefe Addytion of diuers summes, procedinge by some certaine and reasonable ordre.

You shall also vnderstande, that there are infinite kindes of Progressions, but for you (as yet) two are sufficient to be exercised in, of whiche the one I call Arithmeticall, and the other Geometricall.

Arithmetical progressiõ is a rehersing or placing down of many nũbres, nũbre after nũbre, in suche sorte, þ betwene euery two nexte numbres rehersed or placed downe, þ difference, diuersitee, or exces, be equall and a like. Arithmetik progression

Sc. Sir I thank you for that you haue bothe opened vnto me what Progression is, truly, & also why it is here placed. But, I praie you with an exaumple make plaine your definitiõ.

M. Examples can not wante, seing all reasonable creaturs naturally vse the ordre of one kinde of Arithmeticall progression (whi-

the

che therfore is also named Naturall (whā soeuer they distinctly do counte or numbre any multitude on by on, sayinge, 1.2.3.4.5.6. whereby the proceedinge from numbre to nūbre and euery one surmountinge and exceedinge his felowe nexte before by a like quantitie (which here is 1)declareth the same to bee arithmeticall progression . And for the more plaines I set it downe in this manner.

The common exces. 1 1 1 1 1

The progression 1 2 3 4 5 9

Sc. This is moste euidente. And I thinke that I am able to tell you now of any progression Arithmeticall propounded, what is that common exces or difference whereby yt procedeth if this ordre be kepte in it.

M. What say you of 3.6.9.12.15?

S. Thei exceede eche other by 3. And that maye I set downe in such euident ordre , as you did your example of naturall progressiō. in this wyse.

The common exces. 3 3 3 3

The progression. 3 6 9 12 15

Ma. And doe you not allso nowe perceaue that the whole table of Multiplication may bee made by the ordre of progression Arythmeticall? either if ye will beginne at the firste numbre of any of them on the left hande, and so proceede ryghte ouerthwarte ; or at any of the

the firste numbres of the vpper rowe, and go directly downewarde?

S. I pray you let me considre the thing a little, and I will answer you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	90	90	100

By this triall I perceaue it now very well, for the common exces or dyfference betwene any two nexte, is continually as muche as the firste numbre of euery rowe, eyther from the lefte hande ouerthwart taken, or from any of the vppermoste ouerthwart rowes downward,

M. Nowe than if of any suche progression you woulde spedily knowe the totall summe, muche quicklier than by common additions rules: firste tell how many numbres there are (whiche numbres here we cal places or partes) yf thei be odde, write their sū downe by it self, as in this exāple: 2, 4, 6, 8, 10, 12, 14.

1 4, where the numbres are 7 as you maye see, therefore sette downe 7 in a place alone,
7 as I haue done in the margente heere: then adde together the firste numbre and the laste, as in this example: adde 2 to 14, and that maketh 16, take halfe of it and multiplye by the 7, which you noted for the numbre of the places, & the summe that amounteth, is the summe of all those fygures added together, as in this example: 8 multiplied by 7, make 56, and that is the summe of all the figures.

Sc. That will I proue by an other example. I woulde knowe howe muche this sum is, 5, 8, 11, 14, 17, 20, 23, 26, 29. I tell the places, and thei are 9, ỷ I note. Then I put the first numbre 5, and the laste 29 together, and they make 34, I take the halfe of it, that is 17, & multiply by 9, and it maketh 153. That you saye is the summe of all the numbres.

M. So shall you finde it, if you trie it.

S. Howe shall I trie it?

M. By addition: for yf you adde all the parcels togyther, you shall see the same sum amount, if you did worke well. And that manner of Addition trieth all kindes of summing any Progression.

S. Then can I sum a progression, if ỷ nũbres of the parcels be od. But what if they be euen as in this example, 1, 2, 3, 4, 5, 6, 7, 8.

Ma. When the numbre of the parcels ỷ be euen

euen, then note that also as you dyd before, and lykewaies adde the fyrste summe to the laste, and by the halfe of the numbre of the places do you multiplie it. as in youre example, the parcels are 8, that note I then addyng the first sum to the laste, there amounteth 9, that we I multiplye by the halfe of parcels, that is by 4, and it maketh 36, whiche is the summe of the 8 parcelles.

But if you will take one rule for these bothe, do thus. Multiplye the half of the one by the other whole, and the sum wil amounte al one for sometime it chaunceth, ꝥ the numbre of the parcels be odde, so that their halfe can not be taken: and somtime it chanceth the addition of the first numbre & the laste to bring forth an odde numbre, so that the halfe of it can not bee taken: but thei will neuer be bothe odde.

Sc. Then I perceaue this, if there bee no more longing to it.

M. As accustumably it hath byn taughte, this hathe byn the chiefe and onlye exercise in progression vsed. But that you may perceiue how diuers waies & to how greate profite so simple a thinge, as this Aritemeticall progression is, may be considered and vsed, I will here propound you sixe propositions, of whiche foure of them were inuented by a friende of myne, and neuer before this published: and the firste twoe, were neuer to my knowledge written of, but by thre men.

Scholer.

Scholer. Thys doothe greatlye encourage me to be attentiue vnto your wordes, seing I shall not onely be instructed at youre handes in the common knowen rules of thys excellent arte, but besides that so abundantly in other newe rules informed as my verye entraunce shall seeme to passe a greate manye mens farder studie and longer continuance. Therfore Syr I beseche you, let mee knowe youre sixe propositions.

Maister. These they are.

1 To know the last numbre without proceding by continuall addition, till you come vnto it, so that the commō exces, the fyrst numbre, and the numbre of the places be knowne.

2 The firste numbre of the progression and the laste beinge knowne, with the common exces, to finde the numbre of the places.

3 The exces being giuen, and the first or last, to know the quantitie of any middle numbre, whose place is giuen from the first or laste.

4 The totall sum being giuen, and the first and laste, to finde out the numbre of the places.

5 The totall sum of any arithmeticall progression being giuen, and the first and last to finde out the common exces.

6 The totall sum beinge gyuen, and the mutuall exces, with the numbre of the places, to giue the first or last numbre of the same progression.

Maister.

Many mo cõsideratiõs coulde I propoũde you in these Arithmeticall progressions, but these are sufficient to gyue you occasion to think, that rules of knowledge and artes are infinitely capable of enlargement.

Sch. Happy were I, if I did but wel vnderstande ẏ whiche is all readye inuented and written. And yet in my simple fantasye these thinges offer them selues (in manner) to bee studied for about progression. therefore I pray you to procede to the rules answeringe to these propositions.

Ma. I will ordrely for euery of these syxe propositions giue you rules, and with euery one an example, vnleast the plainesse & easynes nede no farder exemplifying?

For the Solution of the firste, Multiplye the exces by a number lesse by 1, than the number of the places, and the ofcome adde to the firste numbre, so shall you haue the laste number, which is sought for.

As (for example), if there were seuen places in a progression arithmetical, whose continuall encrease or mutuall exces, where 4, & the firste numbre were 5, and I wolde know what the last and seuenth numbre is, I multiply 6, which is 1 lesse than 7, the numbre of the places by 4, therof commeth 24, which I adde to 5, that maketh 29: and that is the last numbre, whiche I desired to knowe. And this you may straighte way proue, by

continuall procedinge from 5 til the seuenth place, encreasing euery one by 4, as thus.

5 9 13 17 21 25 29

Loe here, the laste, being also the seuenth, is 29.

Sch. I perceiue all ready one good pro-perti in this rule, whiche in all workes is to be desired: that is, it will ease one from greate labour, yf a progression were propounded of a hundred or twoo hundred places, or moe. And also it is very easye to worke and moste necessarye for the total summe findinge in a very longe progression.

2 Maister. The seconde rule is this. From the laste subtract the firste, the remainder di-uide by the common exces, to the Quotient adde 1, and you haue the number of the pla-ces, whiche you woulde knowe, as in this pro-gression. 6 11 16 21 26 31.

If I knowe onely 6 and 31. and that they encrease by 5, Than according to the rule, from 31 I subtract 6, there remaineth 25, whych 25 I diuide by 5, (the commen exces) the quotient commeth forthe 5, to whiche I adde 1, that maketh 6: and so manye are the places, as you see. Schclar. This rule is so easye, that I were muche to blame, if I coulde not remembre it.

Mayster. The thirde proposition may alwayes thus be soluted. Multiplye th

exc

ceedes by a number lesse by 1, than the distance of the place is from the firste or the laste number gyuen, the ofcome adde to the fyrste, yf the distaunce be reckened from the fyrste, adde the firste also knowne, or subtract from the laste if the distance bee from the laste counted, and the laste gyuen also and that whiche commeth forth eyther in that addition to the firste, or subtraction from the laste, is the numbre soughte. As for example. I propounde you thys Progression.

8 15 22 29 36 43 50 57.

And for the apte consideringe the manner of thys question, I will note ouer euerye place hys distaunce from the firste: and vnder euerye place his distance inclusiuelye from the laste, thus.

1	2	3	4	5	6	7	8
8	15	22	29	36	43	50	57
8	7	6	5	4	3	2	1

Now if the exces wherby this progression standeth bee knowne to bee 7, and the firste number gyuen, beynge 8, I woulde knowe what number standeth vnder 4, that is to saye in the fourthe place, I multiplye 7 by 3 (whiche is lesse by 1 than the number of the place propounded) that yeldeth 21, to whyche (I adde 8, the firste numbre) so commeth 29: which

I saye to belonge to the fourthe p[la]ce, as ye see in the example, it also dothe. Or yf in the thirde place from the laste, you would knowe what numbre in thys example shoulde stande, the laste numbre beynge knowen to be 57, and the common exces 7. than by 2, (which is les by 1 than the place propounded) I multiply 7, that giueth 14 which I subtract from 57, so remaineth 43 which appertaineth to the third place inclusiuely reckened from the laste. and so my example giueth you.

Scholar. I perceiue right good vs[e] of this rule: for if I had forgotten what th[e] firste number were, and remembre stil bu[t] the laste, ye common exces and the numbre o[f] the places, than might I come by ye knowe[-] ledge of my first numbre againe.

And me thinketh, that it differeth n[ot] much from the firste propositiō, sauing th[at] which you make here a middle nūbre, the[re] was made the last: and also in this point [it] differeth, that in it the laste was only soug[ht] and no consideration had in numbring t[he] places from the laste, as here I mar[ke] in your numbres noted vnder your p[ro-] gression.

Maister. And thinke you not, the myd[dle] numbres of a progression standynge o[f a] hundred or three hundred places or m[ore] may as muche cumber a man to comme to [the] knowle[dge]

knowlege of them by cõtinuall encreasinge from the firste, (by the common exces), or abating from the laste continually (the common exces) as the very small numbres in a shorter progression wold do?

Scholar. Yes sir that I think right wel, and therfore I am gladde of this newe framed proposition, and the manner of the working of it.

M. The rule for the fourth is this: Adde ẏ first & the last togither, & by ẏ ofcome diuide the total summe. Double the Quotient, and that will be the number of the places.

Sch. Than if in a progression whose sume were 207, and the first numbre 12 the laste 57, if I adde 57 and 12 together, ẏ maketh 69; and by it I diuide 207, the Quotiēt wil be 3, whiche I double, & so I haue 6. and so many muste be ẏ numbre of the places that this progression standeth on.

M. Whether it be so or no, how wyll you trye?

Sch. halfe 6, which is three, beyng multiplied by 69, must make 207 the total sum. If 6 be the numbre of the places. For so the whole worke of your rule in summing any arithmetical progression did enforme me. I will than multiplie 69, by 3, thus.

69
3
207

It commeth forth iustlye

M. I must much herin commende your

 promp=

promptnes, both in memorye and in wel applyinge youre rule: although in many (est wordes it did containe no such matter.

Sch. Syr I pray you heare mee frame one example more.

M. I am well pleased, so that ye be shorte, for you make me more longer here, than willyngly I woulde haue byn: but I can not perceiue howe I coulde haue omitted anye thing as yet, without youre great lack therof.

Sch. If I had receiued 85 pounds of certaine men, but of howe manye I haue forgotten, yet I remember that the firste gaue mee 7 li, and the last 27 li, and euery payměnt after other did reste by a lyke summe. And the man for whome I receiued thys money, condicioned with mee, that of euery paimente I shoulde haue twelue pence for my laboure, now unleaste I can by arte finde the truthe of this case, I am lyke to lose the most parte of my rewarde.

M. I perceiue you can handsomlye frame an example, which shold concerne your owne gaine. I pray you let me se howe you wold do Justice in this point.

Sch. I adde the first and the last together, that maketh 34: by which I diuide 85, thus.

	1	
	~~27~~	$2\frac{17}{34}$
	~~85~~	
	~~34~~	
	8	

Why, how now? Sir here is a remnant of 17, in whiche 34 can not be had, so that now

nowe I am in the bryers for doublinge of my quotient : and farewell than bothe my Justice and a good lompe of my gaines.

Maister. Ye are neuer the farther from the matter, though yt fall into a fraction. for you shall understande, that the fraction which of any suche worke proceedeth, is euer half of one such, as the vnities of the Quotient before are. And that you may trye, if you double that whiche so remaineth : for than yt will bee equall to youre diuisor, as if ye double 17 (the remenante) yt maketh 34, and youre dyuisour also was 34. thys noteth the remaynder to bee halfe of one.

Scholar. Nowe I am glade of this harde example. For with it I haue a generall rule for the fraction that may happe in this work. So that the Quotiente beinge twoo and a halfe, I double that, and it maketh fyue, therefore shoulde my gayne bee fyue shyllinges. And to bee sure, (by your leaue) I will trie it, for I wil muultiply half of 34. which is that first and last number ioined together by 5, thus.

$$\begin{array}{r} 17 \\ 5 \\ \hline 85 \end{array}$$

It is moste true (I see) that I shoulde leese nothinge by the former workyng.

Maister. The fifte proposition hathe thys rule appertaininge vnto yt, by the fourthe

rule finde the numbre of the places . that beynge done, from the laste subtracte the fyrste, & the residue diuide by a numbre lesse by 1, than the numbre of the places, & the quotient wyll shew the exces, which is sought for.

An example hereof shalbe this: If ye had disboursed 685 poundes to a certaine numbre of men, you neyther can tell howe manye they were, or how much the ones monye exceded his next before, but you are sure that the exces was equall betwene euerye two next: and also you remember that the fyrste had 19, & the laste 118 pounds, how wold you finde both the number of the men and the exces, continually obserued in the succession of their paiments?

Scho. Your rule doth plainly bid, first to find the numbre of ye places, which I will doe according to the fourth rule. I adde 19 and 118 together, thus.

118
19
137

By this 137, I diuide 685, thus.

~~1~~~~3~~
~~6~~~~8~~~~5~~ (5
~~1~~~~3~~~~7~~

Seing there is no fraction, but a whole numbre, beinge 5, I double that: and than muste the numbre of the places be 10. Nowe from the laste I subtract the firste, as 19 from 118, thus.

118
19
99

And so remayneth 99.

This 99 I diuide by a numbre lesse by 1; than the numbre of the places

an

and seinge the place were 89 (11
10, I diuide 99 by 9 thus 89
the quotient is 11, & so was the exces. If I
haue folowed your rule right.

M. You haue wrought euery part of this question bothe well in ordre and trulye in the practise of your rules.

Sc. I will than set it down also formably, so that the numbre of the places, the exces & the total sum may streight appeare, as youre firste exemple stoede.

11 11 11 The com mō exces.
19 30 41
11 11 11 11 11 11 11
52 63 74 85 96 107 118 The progression.

That the places be 10, and that from the first to the last the cōmon exces is 11, I perceiue moste euidently, but whether the total sum be 685, I haue not yet proued, whyche I will nowe do. I adde 19 & 118 togither, that maketh 137: I multiplye ŷ by halfe the nūbre of the places, thus.

137
5
685

All thinges agree moste exactly, so that I am perfect ynoughe in these rules, yf I forget them not againe.

M. Use maketh all thinges perfect.

Your syxt rule is this. By the numbre of "
the places dyuide the total summe, dubble the "
quotient and that wil be the first and the last "
ioyned in one sum. Than by a numbre lesse "
by 1, than the numbre of the places, multy- "
plie

ply the exces, that ofcome subtract frome the
„ fyrste dubbled quotient, and the halfe of the
„ resydue is the fyrste numbre. The laste num-
„ bre you maye diuersly fynde out, as by the
„ fyrste of oure syxe rules, or by subtractynge
„ thys fyrste numbre from the summe whyche
„ here contayned bothe the fyrste and the laste
„ ioyntly, or (thyrdly) by continual adding the
„ exces.

Scholer. I pray you make this somwhat more playne with an example.

Ma. Yf euery moneth in the yeare (counting them nowe as thirten) you gained clerely 40 shyllinges more than you dyd the moneth next going before, and at the yeares ende you finde ÿ whole gaine 5720 shillings, but ye remēbre not how muche either the gaine of the firste moneth or the laste was, by this rule it may be tried out

Sc. So that here ye seeme to apply ÿ 13 moneth to thirtine places, the 40 shillinges every one more thē the other next before it, to be the cōmō exces,
and 5720 s. to be the total Summe.

	~~2~~
	~~2~~ ~~5~~
Master. It is true, by	~~5~~ ~~7~~ ~~2~~ 0 (440
13 than I diuide 5720	~~1~~ ~~3~~ ~~3~~ 3
in this maner.	~~1~~ 1

I double thys quotient, so haue I 880 for the fyrste, and the laste come ioyned togyther by 12, whiche is less

by

by one than the numbre of the places, I multyply 40 (the common exces) so commeth 480. This 480 I subtract fro 880, so remaineth 400: halfe wherof is the fyrste Numbre whyche wee desyred to knowe.

40
12
80
40
480

And as for the laste numbre, I can gyue you yt thre wayes: As by the fyrste of my syre rules, I multiplye the exces by a numbre lesse by 1, than the numbre of the places. as 40 by 12, that gyveth 480, which I ad to ẏ first being 200, so shall the last be 680

The same summe commeth forthe, yf ye subtract 200 from 880.

And thirdely yf I begynne at 200, and so procede encreasinge by 40, I shall at the thyrtenth place haue 680. as thus.

200. 240. 280. 320. 360.
400. 560. 600. 640.
680. 480. 520.

Sch. I thanke you moste hartily for these sixe rules. Nowe if it be youre pleasure I woulde

heare and

and learne somwhat of progression Geome-
tricall.

M. Ther ar yet very many rules & pro-
positions, which fall into this Arithmeticall
progressiõ: but these shal suffice for this time.

And in Geometricall progressiõ I will be
more brife, both bicause I haue ben so long
in this part of Aritmeticale progressiõ, & all
so for that it wolde require the knoledge of
Rootes and numbres surde (wherof ye haue
yet larned nothing) if I should frame the like
propositiõs in them as I haue done in these.
Therfore I will onely teache you two prac-
tises a bout it, and so end the consideration
and workes of these progressions. Progres-
sion Geometrical ys, when the numbres in-
crease by a like proportiõ, þ ys, if þ seconde nũ-
bre cõtaine þ first, 2, 3, or 4 tymes, & so forth,
then the thyrde contayneth the second so ma-
ny tymes also: and so the fourthe the thyrd
and the fifte the fourth:

Progres-
sion Geo
metricall.

wherfore I sett those	3, 6, 12, 24, 48.
three exaumples	1, 3, 9, 27, 81.
Here in the firste ex-	2, 10, 50, 250.

ample you se, that eue
ry numbre contayneth the other that goeth
nexte before him 2 times; and in the seconde
example 3 tymes: in þ third example 5 ti-
mes. Nowe if you wyll knowe how to finde
easily the summe of any suche numbres, do
thus: Consider by what numbre they be mul-
tiplye

plyed, whether by 2, 3, 4, 5, or any other, & by the same nũbre do you multyplye the laste summe in the Progression.

S. I praye you worke it by this example. 2, 8, 32, 128, 512, 2048. which sum I haue encreased by the Multiplication into 4.

M. Thẽ must I multiply the last sum which is 2048 by 4 also, & it wilbe 8192. Now must I bate frõ this sum ỹ first numbre of the progressiõ, which here is 2, thẽ resteth 819 which sum I must dyuide by 1 lesse thẽ was ỹ numbre that I multiplied by. Seynge than I multiplied by 4, I must diuide by 3, so diuiding 8190 by 3, ỹ quotiẽt wilbe 2730, whiche is the summe of all the Progression. And now to prooue whether you can do the same, I gyue you these numbres to adde, 3, 15, 75, 375, 1875, 9375 46875

Scho. I can not well tell by what numbre this Progression doth encrease.

M. In any suche doubte do thus: Dyuyde the second numbre by the firste, and the quotient will shewe you the numbre that engendreth the Progression.

Sc. Then is that numbre in this example 5, for so many tymes is 3 in 15.

M. So is it. Nowe worke as I taughte you.

S. The last numbre is 46875 which I multiplie by 5, and it yeldeth 234375, frõ whiche I bate the firste numbre of the Progression

gression, that is 3, & ther resteth 234371, which I diuide by 4, for that is one lesse the 5, and the quotient is 58593, which is the whole summe of the Progression.

Ma. Now that you knowe the summinge of geometricall progression, I will shewe you a compendious manner either to procede by, or to finde out the quantitye of a nūbre whose distaunce from the firste may bee very greate, whiche to doo by continuall multiplication, woulde be very tedious, if the numbres bee greate and the places many.

Sc. Nothing can pleasure me more than breuity, if it be plaine 14.

M. I thinke I am not yet in anye pointe so dark or harde, that you nede to feare anye obscuritie now. The manner is this: set down of your progression foure or fiue of the firste places, and vnder ye firste put a cyphre, vnder ye second 1, vnder the thirde 2, &c. as if ye had a progression encreasinge by a fiue folde quantitie: as here, 2. 10. 50. 250. 1250. then vnder 2, I putte a ciphre, and vnder 10, ye figure of 1, vnder 50, 2, vnder 250. 3, vnder 1250, 4,

2	10	50	250	1250
0	1	2	3	4

and so forthe yf ye will, but to a wyse and wary woorker, a fewe places were sufficient to proceede by to anye numbre of places in thys sorte, yf any twoo of youre numbres

progressionall

progreſſionall bee multyplyed the one by the other, and the oſcome dyuyded by the fyrſt of your progreſſion, the quotiente is one of youre numbres progreſſionall, and belonginge to that place of youre vnder numbres, that ys equall to that ſumme, that ys made of addition to gyther of youre twoe numbres whiche ſtoode vnder theſe twoe of youre pro-

greſſional nũ- 2 10 50 250 1250
bres that wer 0 1 2 3 4
multiplied the |6210
one by the o- 5
ther, as in thy exaumple.

If I multiplye 10 by 50, therof commeth 500, whiche I dyuide by 2, (the firſt numbre of the progreſſion) and the quotient ys 250: whiche 250 muſte ſtand in the thirde place, bycauſe the numbre whyche ſtandeth vnder 10, ys 1, and that vnder 50 ys 2, and 2 and 1 maketh 3. therfore I ſaye that 250 belongeth to the thirde place of this progreſſion as ye ſee alſo heere dothe. Moreouer if I multiplie 50 into yt ſelfe, there of commeth 2500

50
50
00
250
250
2500

that 2500 I dyuide by 2, the Quotiente

is

is 1250: which muste be sette in the fourth place: bicause 2 added to himselfe againe, maketh 4. and in our example 1250 occupieth the fourth place.

Sc. Than for that fifte place I multiply the progressional numbres ouer 2 and 3, one by the other, and for the syxte, I multiply that ouer 3 in it selfe. &c.

Mast. Ye must well remembre that these places that wee nowe speake of, belong to the vnder numbres, for the true places of the vpper numbres is euer one place more.

Sch. That I see the reason of, bicause the vnder nnmbres begyn one after, and against the firste place of my progression standeth a cyphre, so that the 250 which you said before did belonge to the thirde place, I see belongeth to the numbre of 3, among your vnder numbres, but from the true progression beginning, it is the fourth.

Master. You vnderstande me as I meane. Therfore for youre exercise of bothe the rules here giuen for Geometricall progression, I will aske you a question, muche vsed among the cōmon people (as they haue a greate many the lyke) Yf I wolde sell you a horse, hauing 4 shoes, & in euery shoe sixe nailes, with this condition, that you shall pay for the first naile 1 ob. for the second 2 ob, for the third 4, and for the fourth 8, and so forth dubling vntill the laste nayle. Nowe I demaunde of you

A questiō of selling a horse.

you: how much the pryce of the horse woulde amount vnto?

Sc. Seing the horse hath 4 shoes, & in euery shoe 6 nayles, I perceiue here will be 24 places. Yf I could now haue the laste nūber, I would quickly dispatche this question: I wil therefore wt as few multiplications as I can deuise, come to the knowledge of ye last nūber of this progres-siō. In double I set forth thā a few of my progressiō thus.

1	2	4	8	16	32	64
0	1	2	3	4	5	6

If I now multiply the numbers ouer 5 & 6, the one by the other, I shal haue the number of the eleuenth place for the vnder numbers but of the twelft for the vpper numbers, in whiche my progression standeth, and than that of the eleuenth place vnder, if I multiply in it selfe, I shall haue for the 22 place vnder, but for the 23 of that aboue, whiche I multiply by that ouer 1 of my nether places, and I shall haue the 23 of my nether places, and the 24 of the vpper, which is the number I seeke for.

Maister. Mee thinketh you haue forgotten your rule for abbridginge youre multiplications, for in it the ofte me euer of any multiplication is to bee deuyded by the firste of the progression. And you nowe speake of no diuision.

Scho. Sir I nede not, as my progression beginneth nowe, for If I sholde diuide by 1, it maketh no other quotient, than the nūber is, it doth diuide.

M. It is very wel remēbred and noted of you. to your worke than, accordinge to your prescribed mā-ner, which I like well.

64
32
128
192
2048

S. I miltiply 64 by 32, as here. and it maketh 2048, which is the eleuenth place vnder: but the twelfth aboue, and this I multiplye in it selfe, in this manner.

2048
2048
16384
8192
0000
4096
4194304

And this is ȳ 22 place vnder, but the 23 aboue

I multiply this than by 2, as here.

4194304
2
8388608

And this ofcome 8388608, is my foure twentith place, which I haue found now by 3 multiplications.

Than dooe I resort to the rule of sūming this Progression, where I consider ȳ the encrease of this sūme proceedeth by multiplication of 2, and therfore I do multiply the last sūm by 2 also, & it yeldeth 16777216. fro which I abate the first numbre which is 1, then resteth 16777215, which I should diuide by 1 lesse thā I did multiply: but seing ȳ it is 1, I neede not to diuide it, for 1 (as I haue before sayd) dothe nother multiplye nor di-

diuide: therfore I take that sũ 16777215 for the whole sum of ẏ halfe pennies, whiche by reductiõ I finde to be 699050 s, & 7 d, ob, that is 34952 li, 10 s, 7 d, ob

Ma. That is well done, but I thinke you will buye no horse of the price.

Sch. No sir if I be wyse. Yet for my assurãce will I take so muche paine, as to com to this last 8388608 by cõtinual multiplicatiõ by 2 as in ẏ margent here you may behold my worke tyl I haue done.

Ma. Wel, are ye not almoste wery?

Sch. Well fare my shorte rule, for in trouth it hathe more cũning & more ease.

Ma. Well, than answer me to this question:

A Lorde deliuered to a bricklayer a certaine numbre of flodes of bricke, whereof he willed him to make 12 walles, of suche sort, ẏ the first wal should receaue 2 thirdeles of the whole nũbre & the second 2 thirdeles of ẏ that was lefte: and so euery other 2 thirdeles of ẏ that remai-

1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4096	13
8192	14
16384	15
32768	16
65536	17
131072	18
262144	19
524288	20
1048576	21
2097152	22
4194304	23
8388608	24

ned, & so did the bricklayer: And when ye 12 walles were made, there remaineth one lode of bricke.

Question of brike laying.

Now I aske you how manie lode wente to euery wall, and howe manye lode was in the whole?

S. Why sir, it is impossible for me to tell

M. Nay it is verie easy, if you marke it well. Marke well that I sayde, that euerye wall shoulde recaue 2 thirdeles of the sum that was lefte. Nowe take away 2 thirdeles from any sum, and you muste nedes graunte that ye whiche remaineth, is 1 therdele of the sum laste before: example of 9 from which if you take 2 thirdeles, there will remaine 3, whiche is one thirdele of 9. Likewaies from 3 bate 2 thirdeles, and there wil remaine 1

Scholar. This is true, and nowe I perceaue, that the leaste wall had but two lode of bricke.

M. And by ye same reason maie you know howe manie lode every wall had, accordin as this figure folowinge doth shewe, & ly wayes what the whole sum of brickes wa for if you make 12 sumes, multiplyinge by still from the laste remainer, as you see h on the left side of the table, there will appe all the remayners after euerie wall. & if y multiplye the laste of those 12 summes by also, than will that be the sume of the loa whiche were deliuered to the bricklayer.

Ag

1	12	2
3	11	6
9	10	18
27	9	54
81	8	162
243	7	486
729	6	1458
2187	5	4374
6561	4	13122
19683	3	39366
59049	2	118098
177147	1	354294
531441		

Againe, if you do double euerye remainer, as you see at the righte syde of this table, those numbres will shewe the sum of loades that went to eache wall: wherby also you may perceiue, that eche wall was 3 tymes so great as the next lesser.

S. Lo, nowe it appeareth easye inoughe. Now surely I se that Arithmetike is a right excellent arte.

Mast. You will saye so when you knowe more of the vse of it. For this is nothinge in comparison to other pointes that maye bee wrought by it.

S. Than I beseche you sir, cease not to instructe mee farther in this wonderfull cunninge.

Maister.

The rule of propor-tion.

By the ordre of the science (as men haue taught it) ther shold folowe nexte the extractiō of Rootes of numbre, which bicause it is somwhat harde for you, yet I wil let it passe for a while, and will teache you the feate of the rule of Proportions, whiche for his excellencye is called the Golden rule: Whose vse is, by three nūbers knowen, to finde out a nother vnknowen, whiche you desire to knowe, as thus. If you paye for your borde for three monethes 16 shillings, how much shall you pay for 8 monethes?

The Golden rule.

Questiō of bording

To knowe this and all suche lyke questions, you shall considre which two of your 3 numbres be of one denomination, and sett those two, the one ouer the other, so that the vndermoste bee it that the question is asked of. As in my question 3 and 8, be both of one denomination, for they bothe bee monethes, and bicause 8 is the numbre that the questi on is asked of, I set them one ouer the othe and 8 vndermost thus, with such a crooked draught of lines.

3
8

3 16
8

Then doe I set the other num ber which is 16, against 3, the right syde of the lyne, thus.

And now to knowe my question, th must I doe: I must multiplye the lowe most on the lefte side, by that on the right si

an

and the summe that amounteth I muste diuide by the higheste one the left side. Or in plainener wordes thus: I shall multiplye the numbre of whiche the question is asked (whiche is called the thirde numbre) by the numbre of an other denomination (whiche is called the Seconde) and that summe that amounteth muste I diuide by the summe of like denomination, whiche is called the Fyrste. Than for the knowledge of this question, I multyply 8 to 16, and there amounteth 128, which I diuide by 3, and it yeldeth 42, shillinges, and 2 s remaineth, whiche I turne into pennyes, and they be 24 d, of which the thirde parte is 8 d: so the third part of 128 s is 42 s, 8 d. which sum I write at the right hande of the figure against 8, thus.

The third numbre. The second numbre. The firste numbre.

3 ⟋ 16 s.

8 ⟋ 42 s. 8 d.

Hereby I knowe that if three monethes bordinge do come to 16 s, the 8 moneth bordinge will come to 42 s, 8 d. and likewise of any other like question.

But here must you marke, that the fyrste numbre and the thirde be of one denomination, and also the Seconde and the Fourthe for which you seeke, or else be of suche denominations, that you in workinge may bringe them into one. As if a man shoulde aske mee this question.

Question of expences.

Twelue weekes iourneying coste me 14 nobles, howe manye poundes is that in one

 yeare

yeare ? Here you se no two numbres of one denomination, but yet in workinge you maye tourne them into lyke denominations, as thus. Tourne the one yeare into 52 wekes: & the fourth sum will bee nobles, by the order of the working. Then to knowe this questiō multiply the third sum 52 by the second 14, & the sum wil be 728, that diuide by 12, & it will be 60, and 8 remaining, whiche yf you tourne into shillinges, they wil be 53 s, 4 d. which if it be diuided by 12, will yelde 4 s, 5 d. and the third part of a penny: put this 60 nobles (which maketh 20 li) with ye 4 s, 5 d and q, and little more for the sum that answereth to the question, and it is the expence of a yeare, and the sums wil be thus.

12 — 14 nobles.
52 — 60, 4 s, 5 d, q.

A Generall rule.

And take this for a generall rule, that euermore the Third numbre be it that the question is ioyned with, and the firste, the numbre that is of the same denomination, then must the second nedes be that other.

Remembre also that the place of the First numbre is the highest on the left side: and the place of the secōd right against it on the right side: the place of the Thirde numbre is vnder the first, as by those examples you haue seen.

Scholar. This I trust I can doe.

Ma. But and the question be asked thus In 8 wekes I spende 40 s, howe lo

will 1 0 5 shillinges serue mee? Thoughe the ordre seeme vnlike, yet take you 1 0 5 for the Thirde numbre, and 40 beeinge of the same denomination, for the Fyrste, and then 8 for the Seconde. Then multiplye 1 0 5 by 8, and it will be 840, whiche if you diuide by 40, it will yelde 2 1, which is the Fourth numbre, and sheweth howe manye weekes 1 0 5 s will serue, if you spend 4 0 s in eight weekes.

The fygure of this questiō is this: as if you should say: Yf 4 0 s serue for 8 weekes, 1 0 5 serue for 2 1 weekes.

40	8
105	21

Other diuersities there bee of workinge by this rule, but I had leuer that you wolde learne this one well, then at the beginninge to trouble youre minde with manye fourmes of working, syth this way can doe as muche as al the other: and hereafter you shall learne the other more conueniently.

But yet before wee make an ende of this rule, this shal you note, that there is an other ordre quite contrarye to this that you haue learned. For in this rule hitherto, euermore looke how muche the Thirde number is greater then the Firste, so muche the fourth number is greater then the Seconde. And contrary waies: looke howe muche the Firste summe is greater then the Thyrde, (yf it doe chaunce so) so muche is the Seconde summe Note.

greater

greater thē the forth. But there is a contra ry ordre as this: That the greater the third ſum is aboue ẏ firſte, the leſſer ẏ fourth ſum is beneth the ſeconde: and this rule you may call the Backer rule, as by example.

The Backer rule.

Queſtion of byinge cloth.

If I haue bought 3 0 yeardes of clot of 2 yeardes breadeth, & woulde by canuas o 3 yeardes broade to line it withal, howe ma ny yeardes ſhoulde I neede?

Scholer. Why, there is none ſo broade.

M. I do not care for that. I do put this ex ample only for your eaſy vnderſtandinge: Fo if I ſholde put the example in other meaſure it wold be harder to vnderſtand. But now to the matter: If you wolde knowe this que ſtion, ſet your numbers as you did before; bu you ſhall multipli now the firſt numbre by ſe cond, and that ariſeth there of, you ſhall dy uide by ẏ thirde: which thinge if you do here I meane if you multiplie 3 0 by 2, it will b 6 0: which ſum if you diuide by 3, there wi appere 2 0, whereby I knowe that if 3 0 yeardes of cloth of twoe yeardes broade ſhoulde be lined with canuas of 3 yeardes brode, 2 0 yardes of canuas, woulde ſuffiſe, as this fygure ſheweth.

2 Z 30
3 20

And now bicauſe you founde faulte at my example, how ſay you, perceaue you this?

Scholer. Yes ſyr, I ſuppoſe.

Maiſt. Than aunſwer me to this queſti

How

Howe manye elles of canuas of elle breadth will serue to lyne 20 yeardes of Saye, of 3 quarters of a yarde broade? *Another like question.*

Sch. In good faith sir, I can not tell, for I knowe not howe to bringe the summes to like denominations.

M. Than I will tel you: sith there is mention here of quarters, and againe euery one of the measures both elles and yeardes may be parted into quarters, do you parte them so bothe in the breadeth and length, and than put forth the question by quarters.

Sc. Then I shall saye thus. Howe manie quarters of canuas of 5 quarters broade, will line 80 quarters of 3 quarters broade?

Master. Now answer to the question.

S. Fyrst I will set them doune in their forme, thus, for 5 is ioyned with the question, & is therfore the thirde numbre: then is 3 the numbre of the same denomination, I mean because they be both referred to bredth

3 ⟋ 48
5

Nowe I multiplie 80 by 3, and it is 240, which I diuide by 5, and it yeldeth 48. The say I, that 48 quarters of 5 quarters broade, will seffice to line 80 quarters of 3 quarters broade.

M. Turne the quarters agayne into elles and yeardes.

Sch. Then I saye that 9 elles and 3 quarters of a yearde of elle broade, will serue to

lyne

lyne 20 yeardes of 3 quarters broade, as this figure sheweth.

3 Z 80
5 48

Master. This rule is so profytable for all estates of men, that for this rule onely (if there were more but it) all men were bounde highlye esteeme Arithmetike.

By thys rule may a Captaine in war worke many thinges, as I will hereafter instructe you abundantlye, onelye nowe I wi shewe you this example.

Question of prouision, touching an army.

If it shoulde chaunce a Captaine whic hath 40000 souldiers, to be so enclosed his enimye, that hee coulde haue no fres purueyance of vittayles, and that the vytayles which hee hathe, woulde serue that armye but only 3 monethes, howe manye me shoulde he dimisse, to make the vitaile to suffice ỷ residue 8 monethes?

S. As you taughte mee, I sette the numbres thus, sayinge: If thre monethes suffice 40000, to how many wil 8 moneth suffice?

3 Z 4000
8

To know this, I multiply the first nũbre into the secõd 40000, & it yeldeth 12000 whiche summe I diuide by 8, and there wi be in the quotient 15000, which if I d subtracte from 40000, ỷ remainer will declare that he must dimisse 25000: as thys fygu

3 Z 400
8

gure sheweth.

Ma. Well, syth you perceaue now the vse this rule, I will shewe other whiche en- e of the same. And firste the Double rule, whiche is so called, bicause ther is in it dou- le workinge, by whiche thinge onely it diffe eth from this. Te Doble rule.

S. Then by an example I, shall vnder stāde well ynoughe.

M. So shall you: and let this be the exam- le. If the cariage of 100 pounde weighte 0 miles, doe coste 12 d, howe muche will the cariage of 500 weighte coste, beinge cari d 100 miles? Of cariag

S. I pray you shew me the working of it,

M. You must make 2 workinges of it: the firste thus: If 100 pound weight cost 12 d, howe muche wyll 500 pound cost? Set your fi- gure thus.

100 — 12
500

And multipli 500 by 12, and ther of amounteth 6000 which if you diuide by 100, the quotiēt wil be 60, that is the price of 500, for 30 myles.

Then begin the seconde work sayinge: If 0 miles coste 60 pence, howe muche wyll 00 myles coste?

Set youre figure thus.

30 — 60
100

Than multiply a 100 by 60 wherof amounteth 6000, which being diuided bi 30, wil yelde 200

Than

Than you may saie, that so many penny
shall cost the cariage of 500 pounde waig
100 myles, after the rate of 12 pence fo
100, caried 30 myles.

Scholer. Nowe I perceiue yt also.

M. Then answer me to this question.

Question of sowing

30 busshels of weate, sowede, yeldeth in o
yeare 360, how many wil 80 busshells ye
in 7 yeare? I meane sowinge euery yeare
those 7, still 80 busshels.

Sc. Fyrst I saye, yf 30 bushels wil yel
360 in 1 yeare, the 80 busshels wil yeld 96
in one yeare. Then for the seconde worke
saye: Yf one yeare yelde 960, then 7 ye
wyll yelde 6720. as these two fygures d
shewe.

30 — 360　　1 — 960
80 — 960　　7 — 6720

Question of corne.

But nowe syr if I set forth 30 bushels
corne to an other man for 7 yeare, agreeyn
so that hee shall sowe euery yeare the who
encrease of the corne, and I at the ende
those seuen yeares to haue the halfe of t
whole encrease, I woulde know howe man
busshels will there amount to my parte, su
posinge the encrease to bee after the rate
the laste question, for 30 bushels in one yea
360

Maist. In suche a question you muste ha
so manye seuerall workinges, as there b
yeares: as for exaumple. In the first
yea

yeare 30 buschels yelde 360: then to know the yelding of the seconde yeare, I muste say, if 30 yelde 360, how manye yeldeth 360 worke by your rule, & you shall finde 4320. Then say for ỹ third yere: if 30 yeld 560, how many wil 4320 yeld? you shal haue 51840: and so euery yeare multiplyinge the whole increase by 360, and diuidinge it by 30, the increase of the nexte yeare wil amounte, as these 7 figures doo ordrely declare.

a		b	
30 — 360		30	360
		360	4520

c

30 360
320 51840

d

30 360
51840 622080

e

30 360
622080 7464960

f

30 360
7464960 89579520

g

30 360
89579520 1074954240

Where I haue set 7 letters for the 7 yea-res, of whiche the firste is set without arte, bicause

bycause that is the increase whiche you do presuppose, and the laste numbre of eche ther doothe shewe the encrease of the yea that it standeth for, whiche the letters do declare: so that the encrease of the 7 year is 1 0 7 4 9 5 4 2 4 0 busshels: how mani qu ters that is, and also howe manye waies y maye by Reduction sone fynde.

Nowe with one question more I wy proue youe. If 6 mowyers doe mowe 45 cres in 5 daies, howe manye mowyers w mowe 3 0 0 acres in 6 daies.

Question of mowinge.

S. If 45 acres do require 6 mowyers, t 3 0 0 acrs requireth 40. Nowe agayne; if daies require 4 0 mowyers, then 6 dayes deth but 3 3 mowyers.

M. Why do you not make mention of t 2 that remaineth in the laste diuision? for t laste parte of the question is wroughte by t Backer rule, where the fyrste numbre 5, multiplied into the seconde, that is 4 0, wh of amounteth 2 0 0, whiche if you dyuide the threde numbre 6, the quotient willbe 3 as you saide, but then will there remayne whiche can not well be diuided into 6 par howe be it, you may vnderstande by the si parte of 2, the thirde parte of one man woorke, whiche you muste put to the 3 3 els you maye saye, that 3 3 woorkemen ende all þ 3 0 0 acres in 6 0 daies, saue tw mennes worke for one daie, or 2 daies wo

for one man: But suche broken numbre called Fractions, you shall hereafter more better perceiue, when I shall whollye instructe you of them.

The rule of Fellowshippe.

But now wil I shew you of the rule of Fellowshyp or Company, which hath sundry operations, accordinge to the dyuers numbre of the company. This rule is sometime without difference of time, and somtimes there is in it difference of time. First I will speake of that without difference of tyme, of which let this be an example. Without tyme.

Foure marchantes of one companye made a banke of money diuersly, for the firste laied in 30 li, the second 50 li, the thirde 60 li, & the fourth 100 li, which stock they occupied so longe, till it was encreased to 3000 li. Now I demaunde of you, what shoulde eche man receaue at the partinge of this money? Question of a Banck.

Sch. I perceaue that this rule is lyke the other, but yet there is a difference, whiche I perceaue not.

M. Then will I shewe it to you. Fyrste by addition you shall bringe all the particular summes of the marchantes into one summe, which shalbe the firste summe in youre workinge by the Golden rule, and the whole summe

of the gaines with the stocke shall bee the seconde summe. Now for the thirde summe you shal sette the portion of eche man one after an other, and then worke by the Golden rule, and the fourth sum will shewe you eche mans gaines: as in example.

The parcels of those foure marchauntes make in one sum 240 li: sette ỹ in the firste place, ỹ gaines in ỹ second, & the firste mannes portion of stock in ỹ third place, thus.

240 3000
30

Nowe multiply the second by the thirde, & it will be 90000, which you shal diuide by 242. & there wil appeare 375 li. thus.

240 3000
30 375

And that is the gaynes for the firste man.

Nowe for the seconde man sette the 50 li. ỹ he broughte in ỹ thirde place, & worke as before: & his part wil be 625 li. as this figure sheweth.

240 3000
50 625

Likewaies for the third mā set his money, whiche was 60 li. & his parte of gaines will bee 750 li, as here appereth

240 3000
60 650

And so for ỹ fourth mā, if you set his sum which is 100 li, his gaines will be 1250 pound, as the proofe will declare.

240 3000
100 1250

S. This I perceiue, but is there any way

to

to examine whether I haue well done or no?
M. That must you do by one common proofe whiche serueth to the Golden rule, and all other ensuynge of the same: and that is thys. Chaunge the standinges of the numbres, and set the thirde in the first place, the fourth in the second place, and the firste in the thirde place, and than worke by the Golden rule. and if you haue done well, the fourthe numbre nowe will bee the same that was the seconde before. As for example. I will take the laste worke which was this.

240 — 3000
100 — 1250

Whiche to examine, I alter as I saide, thus.

109 — 1250
240

Now if I multiply ye seconde numbre by the third, and diuide that that amounteth by the firste, then will the fourth nūbre be 3000, which was the second before, as you see here. which is a token, that I haue well done.

100 — 1250
240 — 3000

But as in a single rule one proofe thus is sufficiente, so in a rule where manye operacions bee, you muste tourne euerye of theym as I haue wone with this one.

S. Then for the profe of ye firste worke of this rule, I shoulde turne the numbres thus.

30 — 575
240

50 Z 625 / 240	And the seconde, thus.

And for the thirde thus. 60 Z 750 / 240
And in eche of them if the working were true, the fourth numbre will be still 3000.

Ma. Well now an other example wyll I put to you, not of gaines, but of losse: for one reason serueth for bothe.

Question of losse.

If three marchauntes in one shippe and of one fellowship had bought marchandise, so that the first had layd out 200 li, the secōd 300 li, & the thirde 500 li, & it chaūced by tempest that they dyd cast ouer board into the sea marchaundise of the valew of 100 poūd, how much shold eche man bear in this losse?

S. If I shall do in this, as you did in the other question, then must I ioyne their three portions together, 200, 300, & 500, which maketh 1000: thā say I, if 1000 leese 100, thē shal 200 leese 20, & 300 shal leese 30, & 500 shall leese 50, as by these three figures it dothe appere plaine.

1000 Z 100 / 200 Z 20   1000 Z 100 / 300 Z 30

1000 Z 100 / 500 Z 50

The rule of felowshyp with time.

M. Thus you perceiue the vse of the rule without time. And that you may as well perceaue the same with diuersitie of time, I propose this example.

Foure

Foure marchātes made a cōmō stocke, which at the yeares ende was encreased to 35145 li. Now to know what shall be eche manes portion of gaines, you muste knowe eche mans stocke & time of continuance. Question of a bancke.

The firste man of these foure laide in 669 li, which he did take from the stocke againe at the end of 10 moneths. The seconde man layd in 810 li, for 8 monethes. The thirde laide in 900 li, for 7 monethes. And ẏ fourth layde in 1040 li, for 12 monethes.

This question shall you examine as you dyd the other before, sauing that where as in the thirde place of the figure you set eche mannes summe alone, here you shall set the same beyng multiplied by the nūbre of their tyme: & likewaies in the firste place of the fygure, you shall sette their whole summes so multiplied by their time, and added into one summe, as thus.

The first mannes summe is 669 li, whiche I multiplye by 10 (that was the numbre of his time) and it maketh 6690. The seconde mans sum 810 li, multiplied by 8 (whiche was his time) maketh 6480, the third mās sum 900 li, multiplyed by 7 (for ẏ was his time) yeldeth 6300. The fourth mans sūe was 1040 li and his tyme 12, multiply ẏ one by the other, and it will be 12480.

These foure summes thus multiplied by theyr time, muste be set ordrely in the thirde

place of the figure: and in the firste place must be set the whole sūme of al foure, whiche is 31950. Now to ende the questiō I say first If 19530 dyd get 35145, what did 6690 get? āswere, 7359 li,

a

as by this figure apereth

31950 31545
6690 7359

Likewaies the seconde man had to his part 7128 li, the thirde must haue 6930 li. And the fourth man shall haue for his parte 13728 li. as these figures dope parcelly declare.

b

31950 35145
6480 7128

c

31950 3515
6300 6930

d

31950 35145
12480 13378

S. This I like very well: but what profe is there of this worke?

Ma. The same that I taughte you for the other. How be it, there is vsed both for thys work & the other also this mānner of profe to adde all the portions togither: and yf they agree to the whole sum, then seemeth it well doone, but this is no sure rule.

S. Yet will I proue it in this example. The four parcels are these whiche if I adde togither, there will amoūt 35145. & ÿ was the whole sum, so is this rule true here.

7359
7128
6930
13728

M. And so will it bee still when the worke is truely done.

But if you like to see it proued false, take 10000 li, from the fourth man, & put it to any of the other 3. and then be you sure þ you haue not done wel, & yet will the proofe alow it, for the Addition wil stil bee al one.

Sc. It must needes be so: but what haue I nowe to learne?

M. There are many other excellente partes behinde, of whiche I will not as nowe make mention, bicause that without the knowledge of Fractions, they can not be dulye be taught, & muche lesse vnderstande. Therefore will I propose to you two or three questions more, wherby you may practise þ better the feat of the rule of Felowship, and so make an end for this time,

But this may not be forgotten, that in all suche questions, if the money bee of dyuers kindes, you must by reduction bringe into one kynde, that is to say, to the leaste value that is named in the question. And likewaies shall you doo, in the time bee of diuers kindes, as som yeares, som monethes, weekes and daies, you shal make al monethes, weekes or daies: according as the leaste name of time in the question is: As for example. Note.

Fyrste in diuersitie of monie. Three companions bought 2000 sheepe, and paide for them Question of sheepe.

them 241 li, 13 s. 4 d. of which sũ one payd 101 li, 10 s. The secõd payd 82 li, 17 s, 10 d. And ỹ third paid 57 li, 5 s, 6 d. How many shepe must eche of them haue? Answere. The first shall haue 840. The second 686. And the thirde 474. And that muste you worke thus.

Fyrste consideringe that your money is of dyuers denominations, you shall (by Reduction) bring it al into the smallest denominatiõ which is in it, that is to say, pẽce, & so wil the totall sum be 58000 pẽce. Now if you turn eche mans money into pennies also, the firste mannes sum will be 24360 pence: The seconde mannes sum 19894 d. And the thirde mans money will be 13746 d.

Now to knowe how many sheepe euery mã shal haue, set the whole summe of money ỹ is 58000 d, in ỹ first place, and in ỹ secõd place set the numbre of sheepe, and then ordrely in ỹ thirde place set eche mans mony, & then multiplying the third & the secõd sũmes together & diuiding ỹ that amounteth by ỹ first, there will appeare the numbre of sheepe ỹ eche man ought to haue, as these 3 figures do shewe.

a

58000 — 2000
24360 — 840

b

58000 — 2000
19894 — 686

c

58000 — 2000
13746 — 474

Scholar

Scholer. Why do you set the mony in the first place, seeing in yͤ questiō you say 2 0 0 0 shepe cost 5 8 0 0 0 d. & not thus, 5 8 0 0 0 d coste 2 0 0 0 sheepe?

M. You remembre, I taught you at the beginning of this Golden rule, that the firste & third numbre muste be of one name & of lyke thinges: & euermore yͤ numbre that yͤ questiō is asked, of must be set in yͤ thirde place. Now is the question plainly this: If foure menne bought 2 0 0 0 shepe for 5 8 0 0 0 pence how many sheepe shall eche man haue?

But seing in this question there ought more respect to be hadde to the sum of mony, than to the sum of the persons, (for in the summes of mony is their proportiō toward the shepe, & not in yͤ numbre of persons) therfore muste we turne the question thus.

If 5 8 0 0 0 pēce bought 2 0 0 0 sheepe how manie didde 2 4 3 6 0 d bie? againe how manie did 1 9 8 9 4 d bie? and how manie bought 1 3 7 4 6 pence?

S. I perceiue it resonable, and so shall I do in all like questions.

M. Euen so. But for easinesse of the worke marke this: When so euer the firste and second nūbre hath ciphers in their first places you may bothe in the multiplication and in the diuision leaue out those cyphres, so that you leaue out like many out of both summes: as in this question, the first nūbre 5 8 0 0 0 Note

hath three cyphres, and so hath the seconde, that is 2 0 0 0: therfore cast awaie their ci-phres. and so will the fyrste numbre be 5 8, and the seconde 2: set theym in their places, and worke accordinge to the rule, and you shall perceiue that it will be all one, sauinge that this is the shorter and easier waye, as these three fygures do shewe.

a

58 — 2
24360 Z 840

b

58 — 2
19894 Z 686

c

58 — 2
13746 Z 474

And this you see is boothe the easier and also the more certaine way to knowe the an-swere to this question.

S. Truth it is as you saie: but sir me see-meth I might aske a farther question here, not onely how many shepe eche man shoulde haue, but also what euery sheepe coste.

M. That question doth not onely belonge to this rule, but maye also bee discussed by Diuision, especially yf the questions num-bre bee one onely: as thus. Dyuide the to-tall summe 5 8 0 0 0 pence, bi 2 0 0 0 (other 58 by 2, omittynge the cyphres) and th quotiente wyll bee 29 pence, that is 2 s, 5 howe bee it, by this rule you maye doe y and beste whan the numbre of the questio doothe exceede 1: as yf I shoulde aske th

questio

questiō, 2000 sheep cost, 58000 d, how muche did 20 coste? Then shall I set my figure thus.

2000 — 58000
20

And doing after ẏ rule, ther will amoūt 580 pence, ẏ is 2 li, 6 s, 4 d. ẏ price of one score.

But if you will vse that easy way that I did teache you, you maie chaunge the fyrste and seconde numbre; thus.

2 — 58
20

Yet nowe one question more will I moue (that you may perceaue the vse of all other like) and so make an ende.

There is in a Cathedrall churche 20 Cannons, and 30 Vicars, those maye spende by yeare 2600 li. but euery Cannon must haue to hys parte 5 times so muche as euery Vicar hath: howe muche is euery mannes portion saye you? Questiō of Cannones.

S. I pray you make the answere your self, so shall I perceaue beste the meanes to answere to suche other lyke.

Ma. In this question you muste doo as in those that haue diuersitie of time, for here is diuersitie of portions: Therefore shal you multiply the numbre of the persons by theire difference of portion: (as you dyd in the other by tyme.) Then muste you multiplye the 20 (which is the numbre of Cannons) by 5, for that is the numbre of their porcion) so wyll

it

it be 100: Then 30, (that is the numbre of Vicars) by 1, (that is the numbre of theyre portion) an it will be 30: put thoſe twoe ſũmes togither, and they make 130. then ſay thus: If 130 ſpēde 2600 li, what may 100 ſpende? The rule ſheweth 2000 li.

Vicars. Againe for Vicars: If 130 ſpende 2600 pou͠d, what may 30 ſpend? Anſwere, 600 li. as theſe fygures ſhewe.

130 — 2600 130 — 2600
100 — 2000 30 — 600

But if euery Cannon ſhoulde haue ſo often times 4 li, as the Vicare ſhold haue 3 li, thē ſhould I multiply 20 by 4, (that were 80) and 30 by 3 (that were 90) and then bothe were 170. Then ſhoulde the figures be ſet thus.

li. ſ. d. li. ſ. d.
170 — 2600 170 — 2600
80 — 1223,10,7 90 — 1376,9,5

But this ſort is to harde for you, by reaſon of the fractions, therefore I wyll let it reſt to that place. And by this rule you ſee what the 20 Cannones maye ſpende, which ſumme if you diuide by 20, you ſhall ſe eche Cannons portion: And ſo of the Vicares, yf you diuide theyr ſumme by 30, the quotient wyll declare euery Vicars portion.

Th

The accompting by Counters.

Maister.

Nowe that you haue learned the common kyndes of Arithmetike with the penne, you shal see the same arte in cou̅ters: whiche feate doethe not onely serue for theym that can not write and read, but also for them that can doe bothe, but haue not at somtimes their penne or tables ready with them.

This sort is in two formes commo̅ly: The one by lynes, and the other without lynes. In that that hath lines, the lines do stand for the ordre of places: and in that that hathe no lines, ther must be set in theire steede so many counters as shall need, for eche lyne one, and they shall supply the steede of the lynes.

S. By examples I shoulde better perceaue your meaninge.

M. For example of — 100000 ————————
the lynes, Lo heere — 10000 ————————
you se vi. lines, whi- ✕ 1000 ————————
che stande for syx pla- — 100 ————————
ces, so that þ nether- — 10 ————————
moste standeth for the — 1 ————————

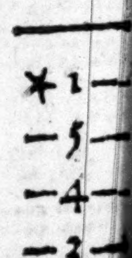
firste

fyrste place, and the nexte aboue it for the seconde; and so vpwarde tyll you come to the highest, which is the syxte lyne, and standeth for the sixte place.

Now what is the value of euerye place or line, you may perceaue by the figures whiche I haue set on them: whiche is accordinge as you learned before þe in Numeratiō of figurs by the penne; for the firste place is the place of vnities or ones, and euerye counter set in that line betokeneth but one: and the seconde lyne is the place of 1 0, for euery counter ther standeth for 1 0. The thirde line, the place of hundredes, the fourth of thousandes: and so forth.

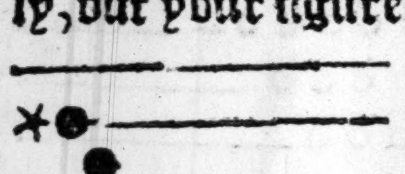
S. Sir I doe perceaue that the same ordre is here of lines, as was in the other figures by places, so that you shall not neede longer to stande about Numeration, except there bee any other difference.

M. If you do vnderstande it, then how will you set 1 5 4 3?



S. Thus as I suppose.

M. You haue set the places truly, but your figures be not meete for this vse: for the meeteste figure in this behalfe, is th[e] figure of a counter, roun[d] as you see here, where I haue expressed that sa[me] summe.

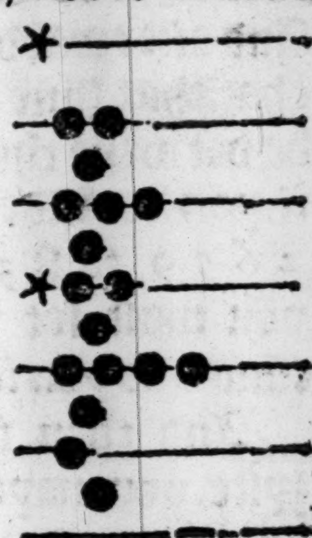


Schola[r]

S. So that you haue not one fygure for 2, nor 3, nor 4, and so forthe, but as manye dygettes as you haue, so manye counters you set in the lowest line: and for euerye 10 you set one in the second lyne: and so of other. But I knowe not by what reason you sette that one cou̅ter for 500 betwene two lines.

Ma. You shall remembre this, that whensoeuer you neede to set down 5, 50, or 500, or 5000, or so forth any numbre whose numerator is 5, you shall sette one counter for it in the nexte space aboue the line that it hath his denomination of, as in this exaumple of that 500, bicause the numerator is 5 it muste be sette in a voide space: and bycause the denominator is hundred, I knowe that his place is the voide space nexte aboue hundredes, that is to say, aboue the third lyne.

And farther you shall marke, that in all working by this sorte, yf you shall set downe any summe betweene 4 and 10, for the first part of that numbre you shal sette downe 5, and then so manye counters more, as there rest nu̅bres aboue 5. And thys is true bothe of digittes and articles. And for example, I will sette downe this sum 187965, whiche sum yf you marke well, you neede



none

none other examples for to learne the numeration of this forme.

But this shall you marke, that as you dyd in the other kinde of Arithmetike set a pricke in the places of thousandes, in this worke you shall set a starre, as you see before.

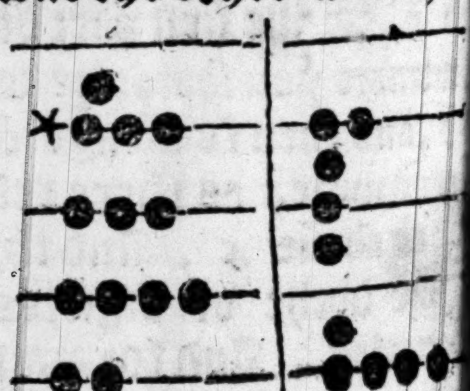
S. Then I perceaue Numeration: but I pray you, how shall I doe in this arte to adde two summes or more together?

ADDITION.

Maister.

THe easyest way in this arte, is to adde but two sumes at ones together: howbeit, you may ad more, as I will tel you anone. Therfore when you wil adde two summes, you shall first set downe one of them, it forceth not whiche, and then by it drawe a line crosse the other lines. And afterward set downe the other summe, so that that lyne maye be betwene them: as if you woulde adde 2659 to 8342, you must set youre sumes as you se here.

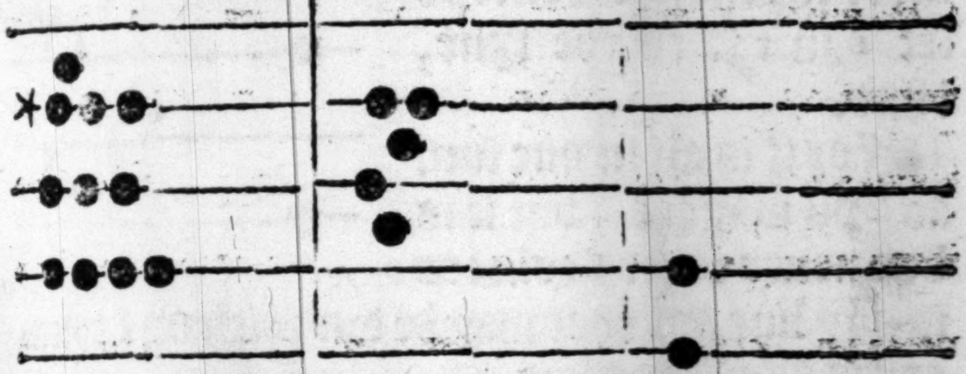
Addition of two summes.



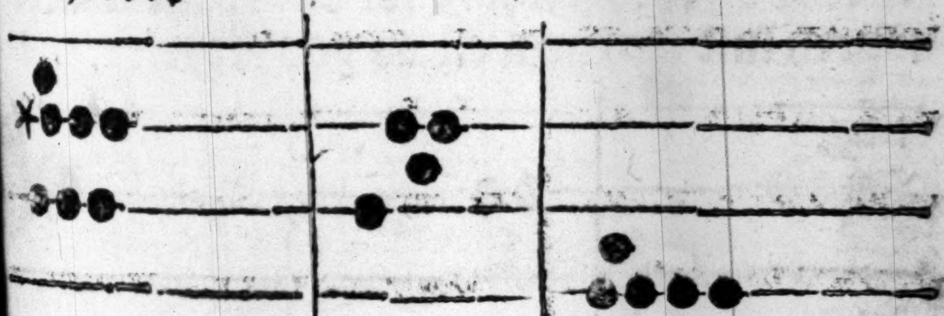
And then if you lyste, you maye adde the one to the other in the same place, or els you may adde them bothe together in a newe place

place: which way, bicause it is moste playnest, I will shewe you firste.

Therefore will I beginne at the vnities, whiche in the firste summe is but 2, and in the seconde sum 9, that maketh 11. Those we I take vp, and for them I set 11 in the newe roume, thus.

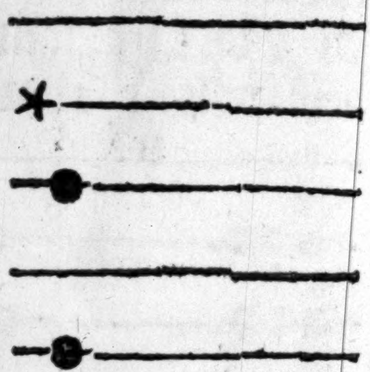


Then I we take vp all the articles vnder a hundred, which in the fyrste summe are 40, and in the seconde sum 50, that maketh 90: or you may say better, that in the first summe there are 4 articles of 10, and in the seconde sum 5, whiche maketh 9, but then take heede that you set them in their right lines, as you see heere.



Where I haue taken away 49 from the

first summe, and 50 from the second, and in their steede I haue set 90 in ye third roume, which I haue set plainely, that you might well perceiue it: how be it, seinge that 90 with the 10 that was in the thirde roume already, doth make 100, I might beter for those 6 coūters set 1 in the thirde lyne, thus.

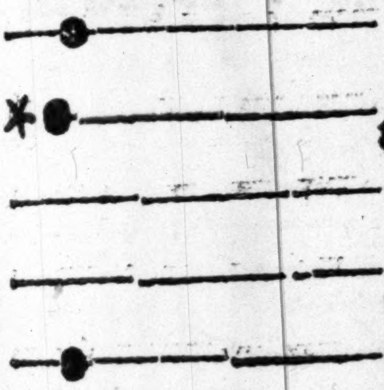


For it is all in one sum as you may see, but it is best neuer to set 5 coūters in any line, for that may be don with one counter in a higher place.

Sc. I iudge that good reason, for many are vnneedfull where one will serue.

M. Wel, then wil I adde forth of hundredes: I finde 5 in the first sum, & 6 in the seconde, which maketh 9 0 0 0, then do I take vp & set in the thirde roume wher is one hundred already, to which I put 9 0 0, and it will bee 1 0 0 0, therfore I set one counter in the fourth line for them all, as you see here.

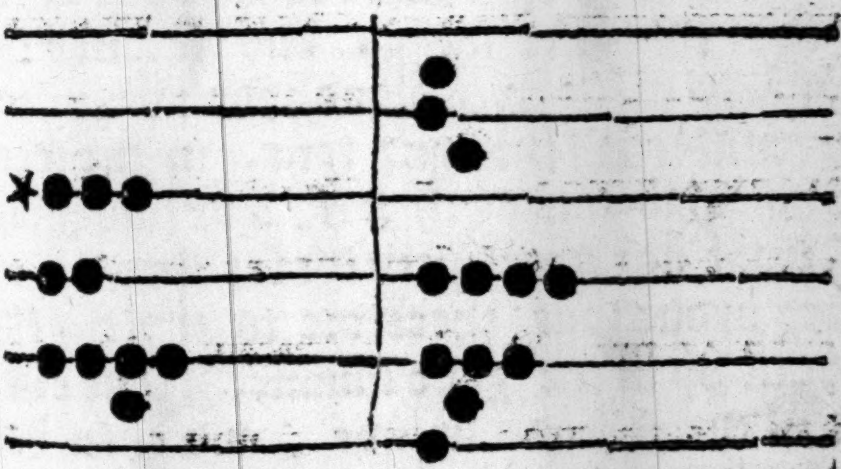
Then adde I the thousandes togyther, which in the fyrste summe are 8 0 0 0, and in ỳ seconde 2 0 0 0, that maketh 1 0 0 0 0 them doo I take vp from those twoe places, and for theim I set one counter in the fyfte lyne, and the appeareth as you see to be 1 1 0 0 1, for so manye doothe amounte of the Addition of 8 3 4 2 to 2 6 5 9



S. Sir, this I do perceaue: but howe shall I set one sum to an other, not chaunging them to a thirde place?

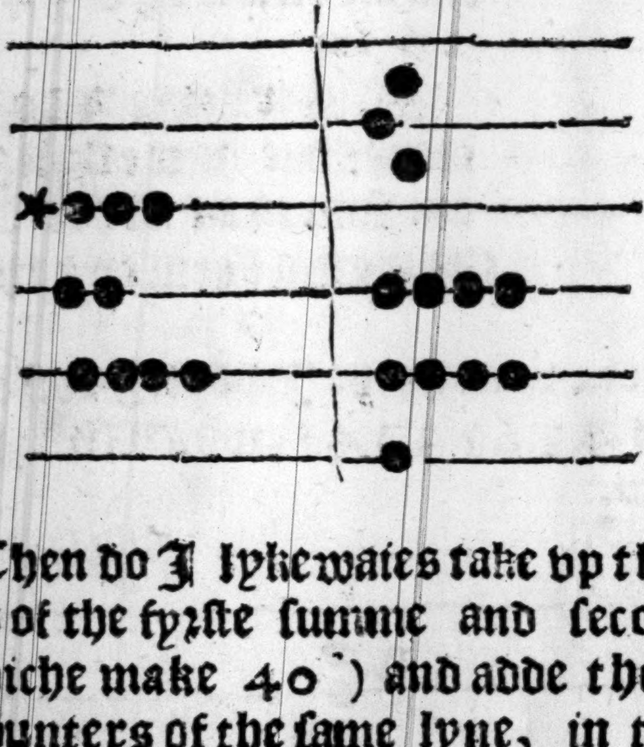
To adde two summes togither.

M. Marke well howe I do it: I will adde togither 6 5 4 3 6 & 3 2 4 5, which first I set downe, thus.

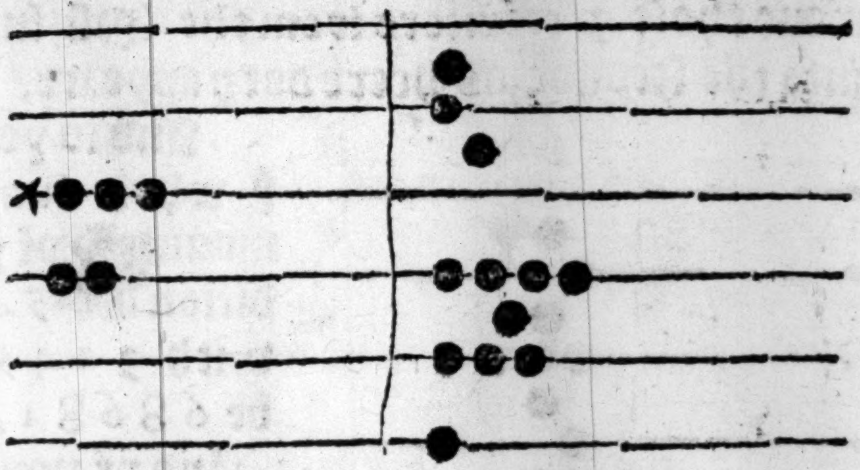


Then dooe I begynne with the smalest, whiche in the firste summe is 5, that doe, I take vp, and woulde put to the other 5 in the

ſeconde ſum, ſauinge yͭ two counters can not be ſet in voide place of 5, but for them bothe I muſte ſet 1 in the ſecond line, which is the place of 10: therefore I take vp the 5 of the firſte ſum, and the 5 of the ſecond, & for them I ſette 1 in the ſeconde lyne, as you ſee here.

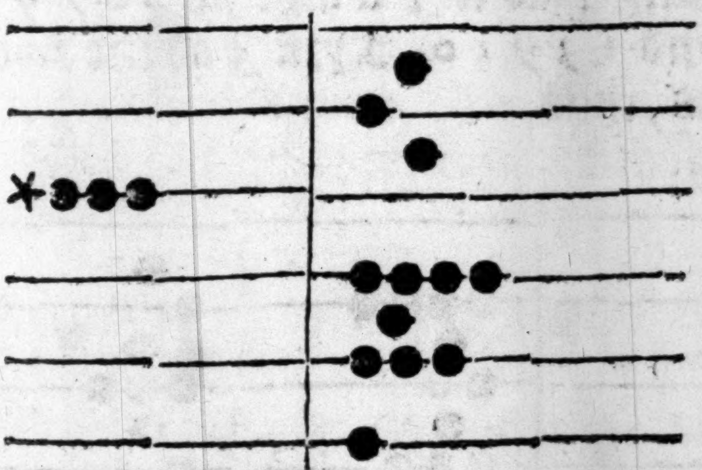


Then do I lykewaies take vp the 4 counters of the fyrſte ſumme and ſeconde lyne. (whiche make 40) and adde theym to the 4 counters of the ſame lyne, in the ſeconde ſumme, and it maketh 80. But (as I ſayde) I maye not conuenientlye ſet aboue 4 counters in one lyne, therefore to thoſe 4 that I tooke vp in the fyrſte ſumme, I take one allſo of the ſeconde ſumme, and then haue I taken vp 50, for whiche 5 counters I ſette downe one in the ſpace ouer the ſeconde lyne, as here doth appeare.



And than is ther 8 0, as wel with those 4 counters, as yf you had sette downe the other 4 also.

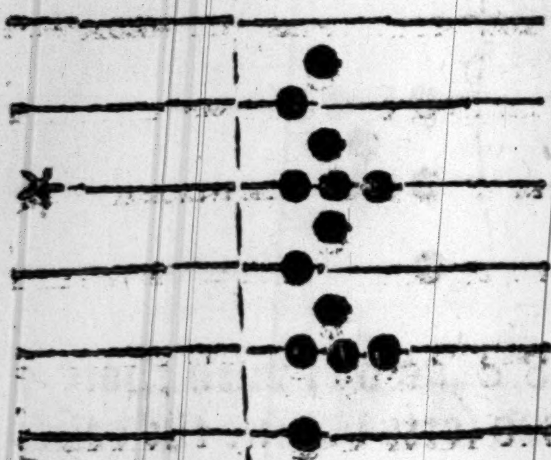
Now do I take the 200 in the first sum, & adde them to the 400 in the second sum, & it maketh 600, therfore I take vp the 2 coũters in the first sum, & 3 of thẽ in ẏ secõd sum, & for them 5, I set 1 in ẏ space aboue, thus.



Then I take the 3000 in the first sum, vnto whiche there are none in the seconde summe agreeinge, therefore I dooe onely re-

 moue

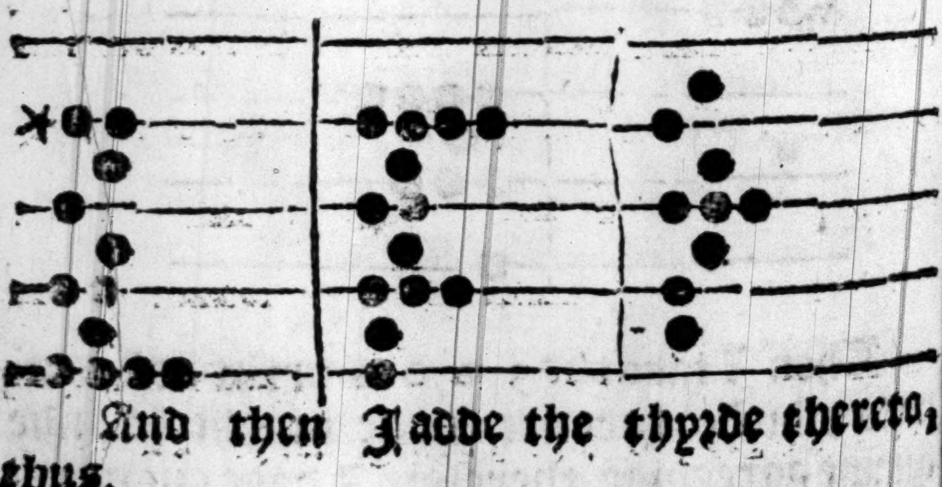
moue those 3 counters from the fyrst summe into the seconde, as heere doth appeare.



And so you see ẏ whole sum ẏ amounteth of ẏ Addition of 6 5 4 3 6 with 3 2 4 5, to be 6 8 6 8 1.

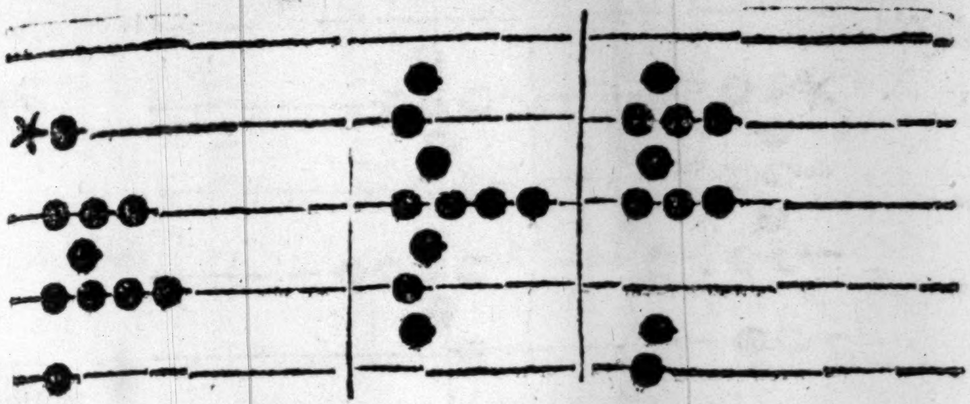
And yf you haue marked these two exāples well, you neede no farther instruction in Addition of 2 only summes: but if you haue more then twoo summes to adde, you maye adde theim thus.

Fyrste adde twoo of them, and then adde the thirde and the fourthe, or more yf there be so many: as if I wolde ad 2 6 7 9 with 4 2 8 and 1 3 9 1 6. Fyrst I ad the two fyrst summes, thus.



And then I adde the thyrde thereto, thus.

And



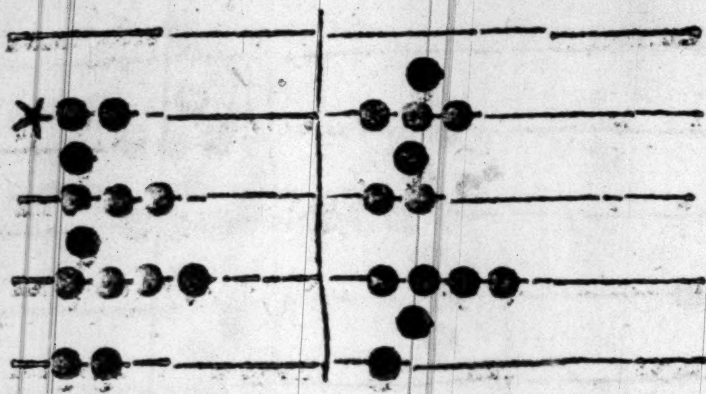
And so of more, if you haue them.

Scholer. Nowe I thinke beste that you passe foorthe to Subtraction, excepte there be any waies to examine this manner of Addition, then I thinke that were good to bee knowen next.

Master. There is the same profe here that is in the other Addition by the penne, I meane Subtraction, for that onlye is a sure waye: but consyderinge that Subtraction muste bee first knowen, I will fyrste teache you the arte of Subtraction, and that by this example.

SVBTRACTION.

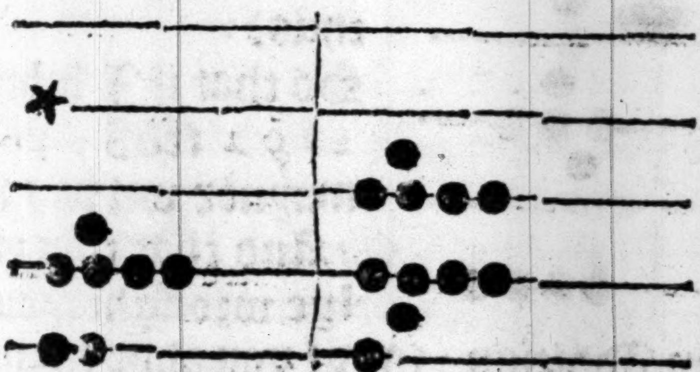
I Woulde Subtracte 2892 out of 8746. These summes must I set down as I did in Addition: but here it is beste to sette the lesser numbre fyrste, thus.



Then shall I begin to subtracte the greatest numbres firste (contrarye to the vse of the penne) that is the thousandes in this example: therefore I fynde amongeste the thousandes 2, for whiche I withdrawe so manye from the seconde summe (where are 8) and so remaineth there 6, as this example sheweth.

Than doe I likewaies with the hundreds, of whiche in the firste summe I finde 8, and in the seconde summe but 7, out of whiche I can not take 8, therfore this muste I doe: I muste looke howe muche my summe differeth from 10, which I fynd here to be

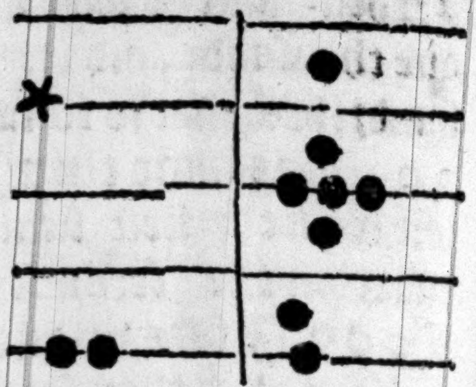
2; then muste I bate for my summe of 8 0 0, one thousande, and set downe the excesse of hundredes, that is to saye, 2, for so muche 1 0 0 0 is more then I should take vp. Therfore from the first summe I take that 8 0 0, and from the seconde summe (where are 6 0 0 0) I take vp one thousande, and leaue 5 0 0 0: but then set I downe the 2 0 0 vnto the 7 0 0, that are there already, and make them 9 0 0, thus.



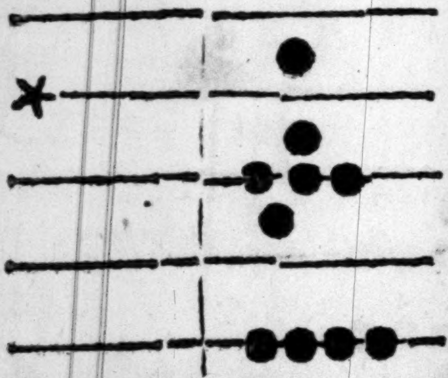
Then come I to the articles of tennes, where in the first summe I finde 9 0, and in the seconde summe but onely 4 0: Now considering that 9 0 can not be bated from 4 0, I looke howe muche that 9 0 dothe dyffer from the nexte summe aboue it, that is 1 0 0, or elles (whiche is all to one effecte) I looke howe muche 9 dothe differ from 1 0, and I finde it to be 1, then in ye steede of that 9 0, I do take from the seconde summe 1 0 0: but considering that it is 1 0 to muche, I sette downe 1 in the next lyne beneth for it,

as you see heere.

Sauinge that here I haue set one coũter in the space in steed of 5 in ẏ next lyne.



And thus haue I subtracted all saue twoo, whiche I muste bate from the 6 in the second summe, & there will remaine 4, thus.

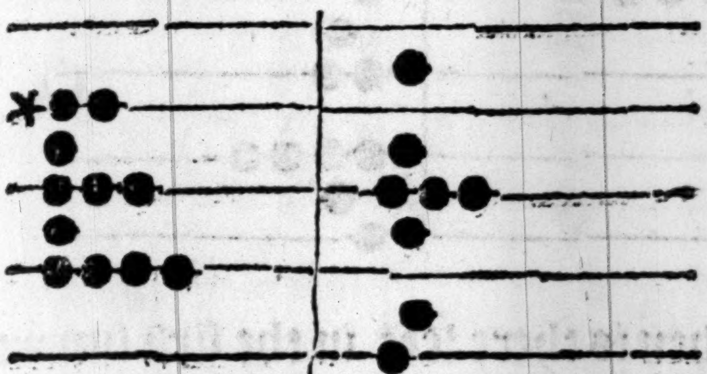


So that if I subtracte 2892 frõ 8746 ẏ remayner wil be 5854.

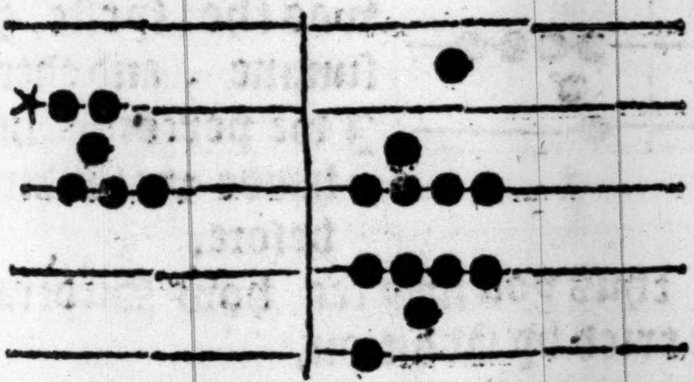
And that this is truly wrought, you may proue by Addition: for yf you adde to this remainer the same summ that you did subtract, than wyll the former summe 8746 amoũt againe.

A profe. S. That will I proue: and fyrste I set the sum that was subtracted, which was 2892, and then the remayner 5854, thus.

Then do I adde fyrste the 2 to 4, whiche maketh 6: so take I vp 5 of those counters, and in their steede I sette 1 in the space, as here appeareth.

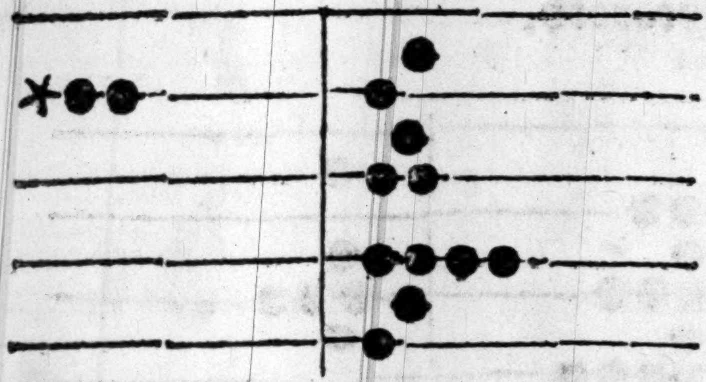


Then do I adde the 9 0 nexte aboue to the 5 0, and it maketh 1 4 0, therfore I take vp those 6 counters, and for theim I set 1 to the hundredes in the thirde line, and 4 in the seconde line, thus.

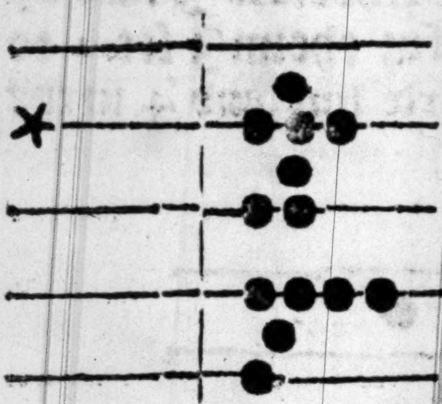


Then do I come to the hundredes, of whiche I finde 8 in the firste sum, and 9 in the seconde, that maketh 1 7 0 0: therfore I take vp those 9 counters, & in their steed I set 1

in the fourth line, and 1 in the ſpace next beneth, and 2 in the third line, as you ſe here.



Then is there left in the firſt ſumme but only 2000, which I ſhal take vp frō thēce, and ſette in the ſame lyne in the ſeconde ſum to the one that is ther already: and then will the whole ſumme appeare as you may well ſe, to be 8746. which was the fyrſte groſſe ſumme, and therefore I perceiue that I hadde wel ſubtracted before.



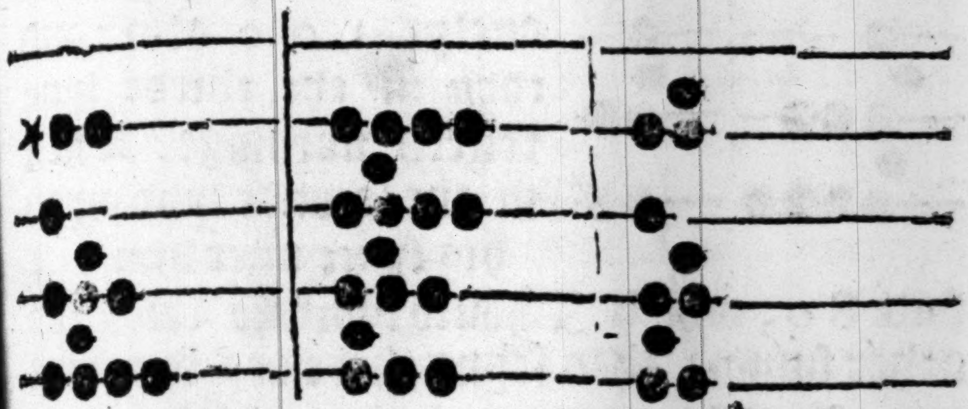
And thus you may ſee how Subtraction may be tried by Addition.

S. I perceaue the ſame ordre here with counters, that I learned before in figures.

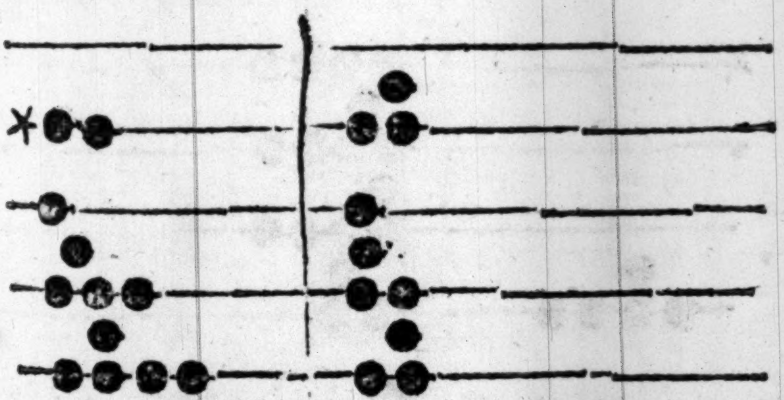
Ma. Then let me ſe, how can you try Addition by Subtraction.

Sc. Fyrſt I will ſet forth this exampl

of Addition, where I haue added 2189 to 4988, and the whole sum appeareth to bee 7177.

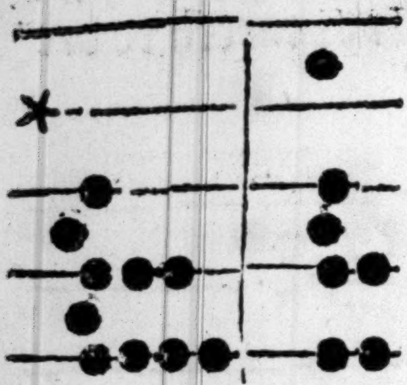


Now to trye whether that summe be well added no no, I will subtracte one of the fyrste two summes from the thyrde, and if I haue well doone, the remayner wilbe like that other summe, as for example: I will subtracte the fyrste summe from the thyrde, whiche I sette thus, in their order

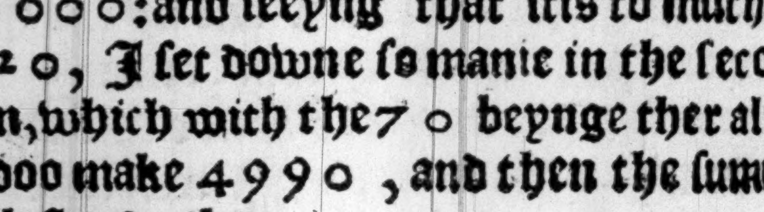


Then do I subtract 2000 of the firste su from the seconde sum, and then remaynethe there 5000, thus.

Then

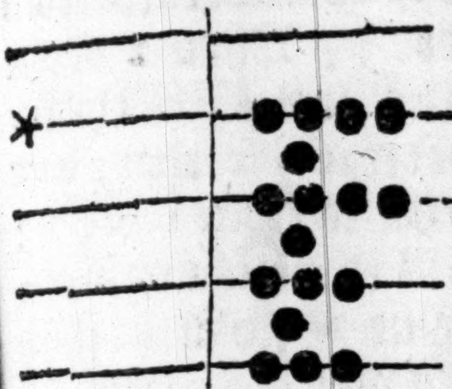


Then in the thirde line I ſubtract the 200 of the fyrſte ſumme from the ſeconde ſumme, where is onelye 100 alſo: and then in the thirde lyne reſteth nothinge. Then in the ſeconde lyne wyth his ſpace ouer him, I finde 80, which I ſhould ſubtract from the other ſumme, then ſeyng there are but onlye 70, I muſt take it out of ſome higher ſum, which is here onely 5000: therfore I take vp 5000: and ſeeyng that it is to muche by 4920, I ſet downe ſo manie in the ſeconde roum, which with the 70 beynge ther alreadye, doo make 4990, and then the ſummes dooth ſtande thus.



Yet remaineth there in the fyrſte ſumme 9 to be bated from the ſeconde ſumme, where in that place of vnities dooth appeare onlye 7: then muſt I bate a hygher ſumme, that is

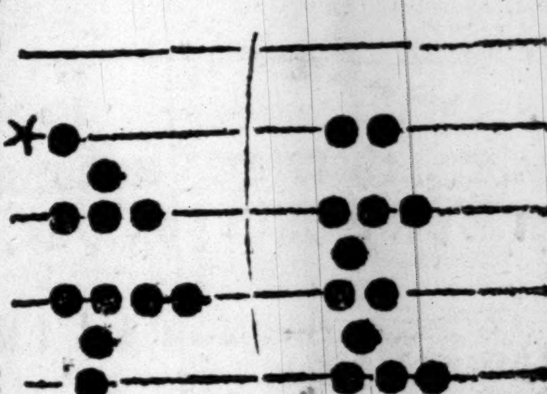
is to say 1 0, but seyng that 1 0 is more then 9 (whiche I shoulde abate) by 1, therfore shal I take vp one counter from the seconde lyne, and set down the same in the fyrste or lowest lyne, as you see here.



And so haue I ended this woorke, and the summe appereth to be the same, whiche was the seconde summe of myne addition, and therefore I perceaue, I haue well done.

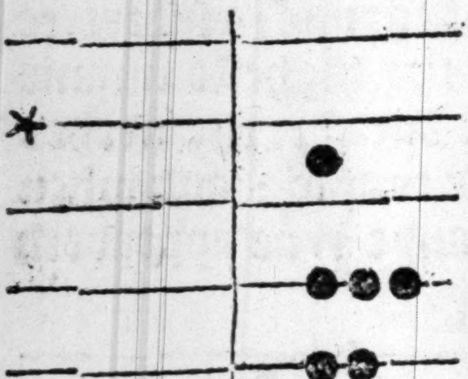
M. To stande longer about this it is but folly, except that this you maye also vnderstande, that manye do begynne to subtracte with counters, not at the higheste summe as I haue taught you, but at the nether moste, as they dooe vse to adde: and when the sume to be abated in anye lyne appeareth greater then the other, then doo they borowe one of the next hygher roume as for example. If thei shoulde abate 1 8 4 6 from 2378, thei set the summe thus

An other way of addition.



Fyrst they take 6 whiche is in the lower lyne, and his space from 8 in the same roumes

mes in the seconde summe, and yet there re-
maineth 2 counters in the lowest lyne. Then
in the seconde line muste 4 be subtracte from
7, and so remaineth there 3. Then 2 in the
thyrde lyne, and his space from 3 of the se-
cond sum can not be, therfore doe theye bate
it from a higher rome, that is from 1000:
and because that 1000 is to muche by 200
therefore muste I set downe 200 in the third
lyne, after I haue taken vp 1000 from the
fourthe lyne. Then is there yet 1000 in ye
fourth lyne of the fyrste summe, whiche if I
withdrawe from the seconde summe, then
dooth all the fygures stande in ordre thus.



So that (as you see) it differeth not greatly whether you beegynne Subtraction at the higher lines, or at the lower.

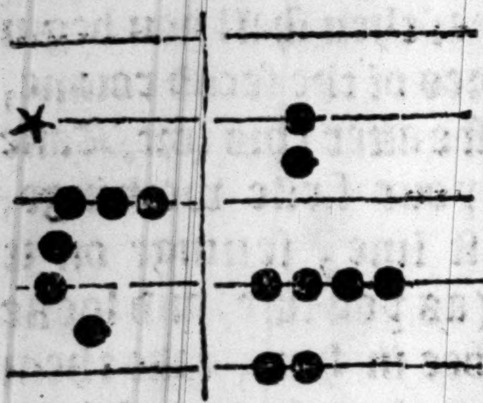
Howe be yt, as some men like the one waye best, so some lyke the other, therfore you nowe knowynge bothe, maie vse whiche you lyste.

MVLTIPLICATION

But nowe touching Multiplicati-
on: you shall set your numbres in
two roomes (as you did in those

two

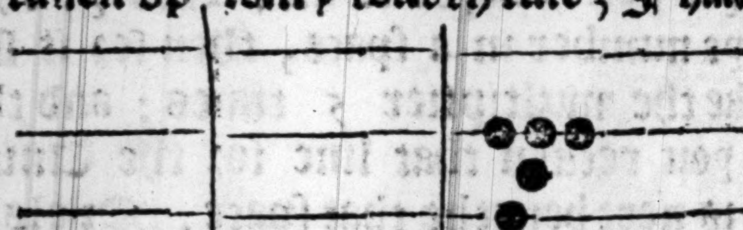
two other kindes) but so that the multiplier be set in the firste roume: then shall you begin with the highest numbres of the secōd roume, and multiplie them firste after this sort. Take that overmoste line in your firste workinge, as if it were the lowest line, settinge on it some moueable marke (as you liste) and looke howe many counters bee in hym, take them vp, & for them set downe the whole multiplier so many tymes as you tooke vp counters: reckening (I saye) that line for the Vnities. And when you haue so done with the highest number, then come to the nexte line beneth, and do euen so with it, and so with the next, tyll you haue done all. And yf there bee anye number in a space, then for it shall you take the multiplier 5 times: and then muste you recken that line for the Vnities which is next benethe that space. Or els after a shorter waie, you shall take onelie halfe the multiplier, but then shall you take the line nexte aboue that space for the line of Vnities. But in suche workinge, yf chaunce your multiplier be an odde numbre, so that you can not take the halfe of it iustelie, then muste you take the greater halfe, and sette downe that, as if that it were the iuste halfe, and farther you shall sette one counter in the space beneth that lyne, whiche you recken for the lyne of vnities, or els onelye remoue forwarde the same that is to bee multiplied.



S. If you set forthe an exaumple hereto I thinke I shall perceaue you.

Ma. Take this example: I wold multiplye 1542 by 365, therfore I set the nũbres thus.

Then firste I beginne at the 1000 in ỳ highest roume, as if it were the fyrste place, & I take it vp, setting downe for it so often (that is ones) the multiplier, whiche is 365 thus as you se here: where for the one counter taken vp from ỳ fourth line, I haue sette

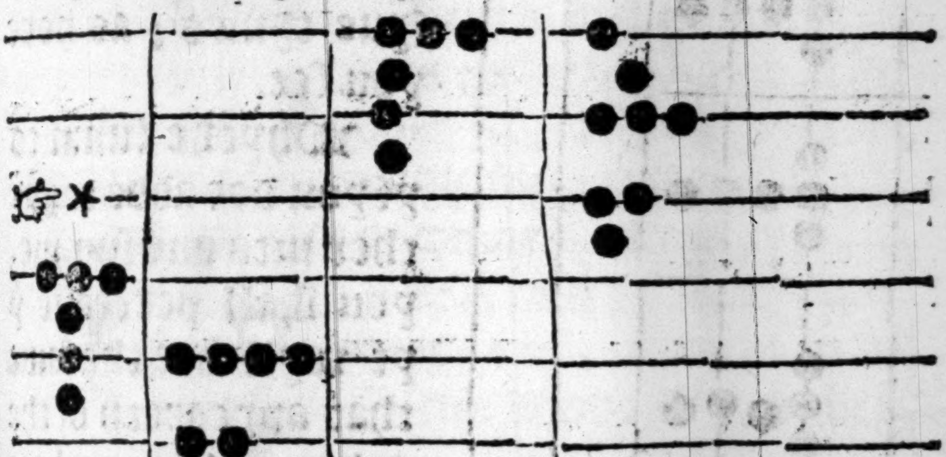


downe other 6, whiche make the sum of the multiplier, reckening that fourth lyne as it were ỳ fyrste, which thing I haue marked by the hand set at the beginning of the same.

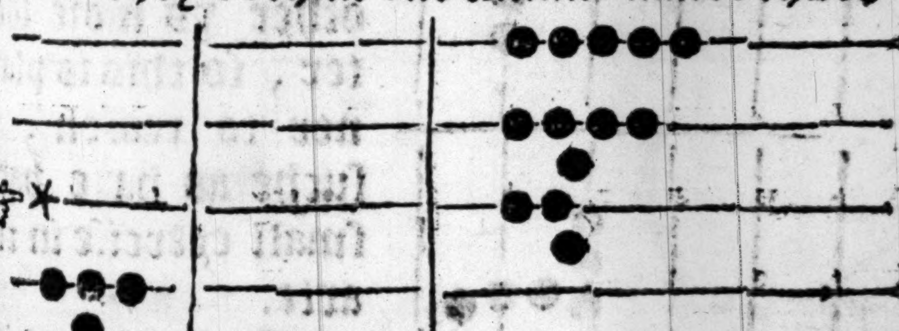
S. I perceaue this well, for in dede this sum

ẏ you haue set doun is 365000: for so much doth amount of 1000, multiplied by 365.

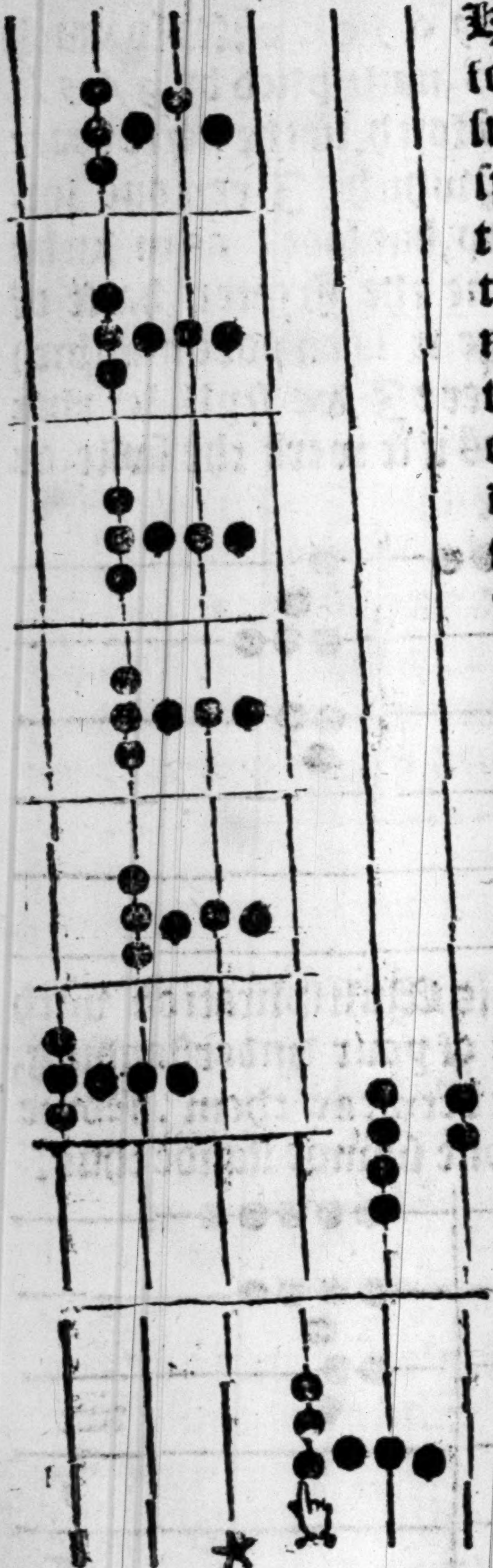
M. Wel, then to go forth, in the nexte space I fynde one counter, whiche I remoue forwarde, but take not vp, but doe (as in suche case I muste) set downe the greater halfe of my multiplyer) seinge it is an odde numbre) which is 182, and here I we styll let that fourthe place stande, as if it were the firste, as in this forme you see.



Where I haue set this Multiplication with other: but for the ease of your understanding, I haue set a little line betwene them. Nowe shoulde they both in one summe stande thus.



An other forme of multiplicaciun.



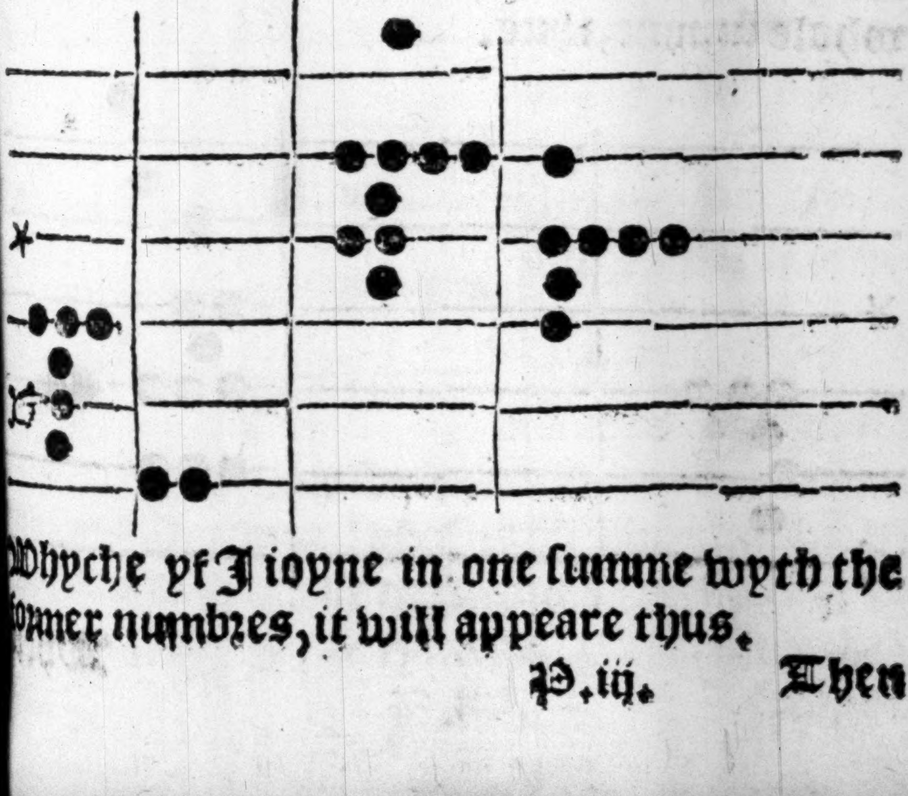
Howe be it, an other fourme to multyplye suche Counters in space, is thys: Firste to remoue the fynger to the lyne nexte beenethe that space, and than to take vp that counter, and to sette doune the multiplier fyue tymes: as here you see.

Whyche summes yf you doe adde togyther into one summe, you shall perceaue ye yt wyll bee the same that appeareth of the other workinge beefore, so that boothe sortes are to one intente: butte as the other ys more shorter, so this is playner to reason, for suche as haue hadde small exercise in this arte.

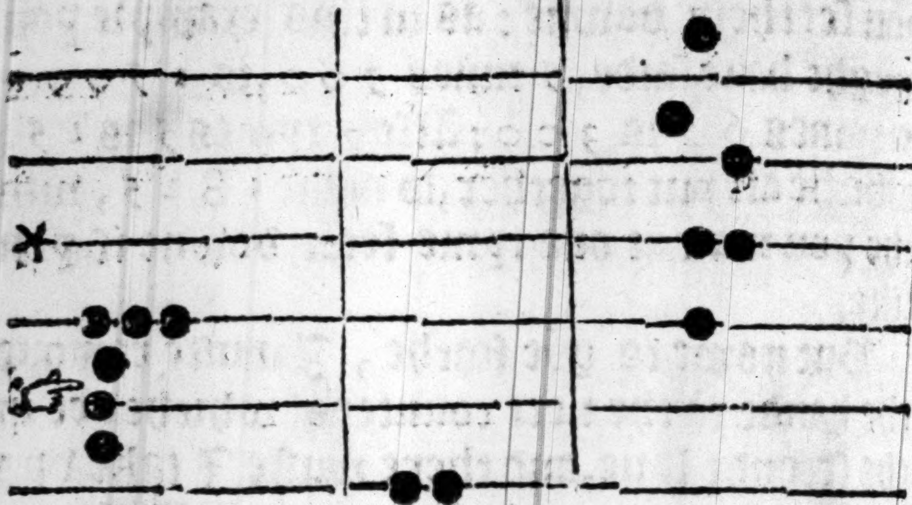
Notwithstanding you maye adde them in your mynde befor ys

you set them downe: as in this example you myght haue saide, 5 times 300, is 1500: & 5 tymes 60 is 300: Also 5 tymes 5 is 25, whiche all put together, do make 1825, whiche you maie at one tyme sette downe if you liste.

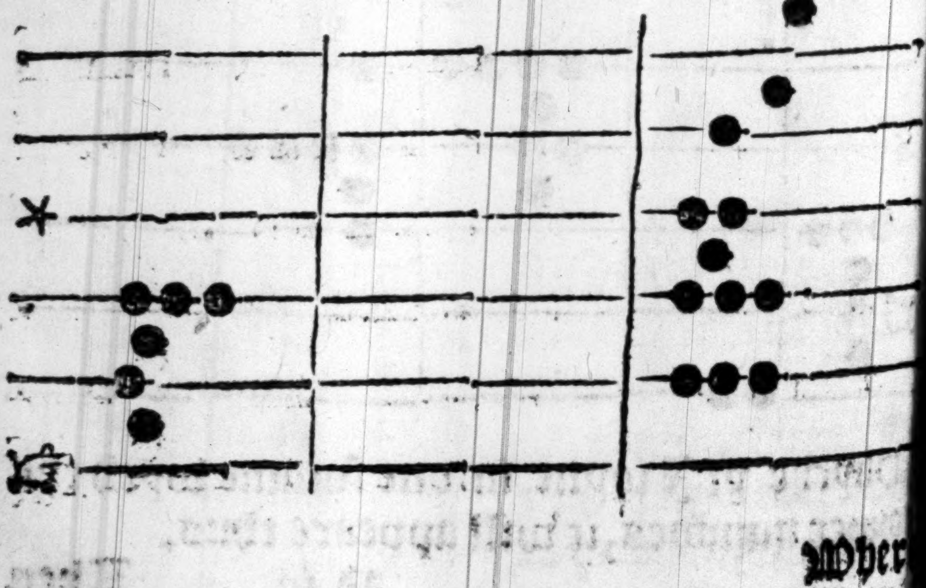
But nowe to goe forthe, I muste remoue the hande to the next counters whiche are in the seconde lyne, and there muste I take vpp those 4 counters, settinge downe for theym in my multiplier 4 times: whiche thinge other I maye doo at 4 tymes seuerallye, or els I maye gather that whole summe in my minde fyrste, and then set yt downe: as to saye, 4 times 300, is 1200: 4 times 60, ar 240: and 4 tymes 5 make 20, that is in all 1460, that shall I sette downe also, as here you see.



Whyche yf I ioyne in one summe wyth the former numbres, it will appeare thus.



Then to ende this multiplication, I remoue the fynger to the lowest line, where are onelye 2, them twe I take vp, and in theire steede to I set down twise 3 6 5, that is 7 3 0, for which I set one in the space aboue ehe thirde lyne for 5 0 0, and 2 more in the thirde lyne, with that one that is there all ready, and the reste in theire ordre, and so haue I ended the whole summe, thus.



Wher

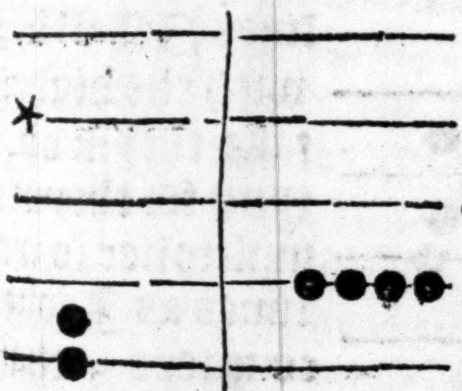
Whereby you see, that 1542 (whiche ys the numbre of yeares sithe Christes incarnation) being multiplied by 365 (which is the numbre of dayes in one yeare) doth amount vnto 562830, wich declareth the numbre of daies sithe Christes incar nation vnto the ende of 1542 yeares, (beside 385 daies & 12 houres for Leape yeares.

The sum of the dayes syth Christe incarnacion.

Scholer. Nowe will I proue by an other example, as this: 40 laborers (after 6d the day for eche man) haue wrought 28 dayes: I woulde knowe what theire wages doth amounte vnto.

A Que of wages

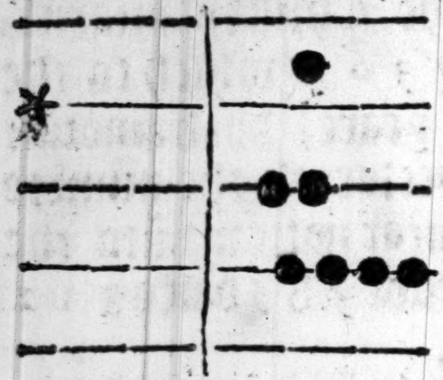
In thys case muste I worke doublelye: fyrste I muste multiplie the numbre of the labourers by the wages of a manne for one day, so will the charge of one daie amounte. Then secondarily shall I multiply ye charge of one daye by the whole numbre of dayes, and so will the whole summe appeare: first therfore I shall set the summes thus.



Where in the first place is the multiplier (that is one daie wages for one man) and in the seconde space is set the numbre of the workemen to be multiplied.

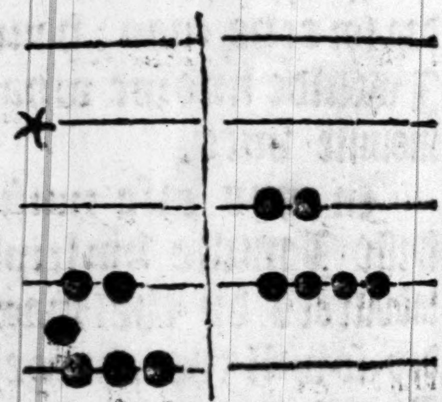
Then saye I: 6 tymes 4 (reckening that

seconde lyne as the lyne of vnities) maketh



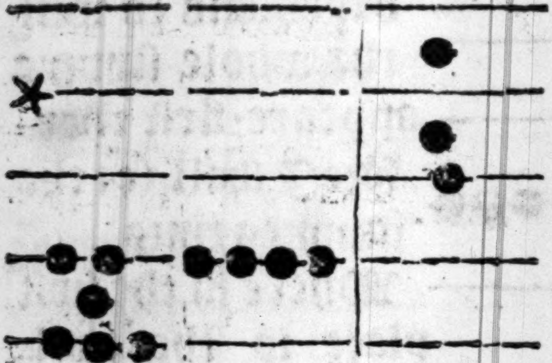
24, for which summe I should set 2 coūters in the thyrde line, and 4 in the seconde, ther fore do I set 2 in the thyrd lyne, and let the 4 stande still in the se, conde lyne, thus.

So appereth ỹ whole dayes wages to bee 240 d, that is 20 s.

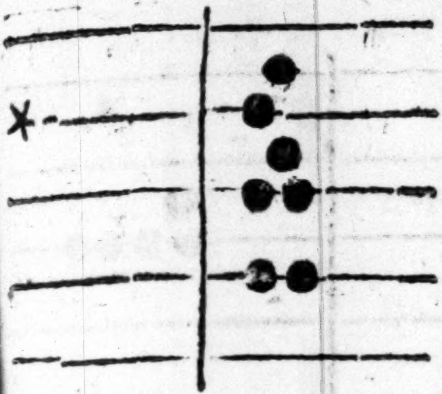


Then do I multiplye agayne the same summe by the numbre of daies, and fyrste I set the numbres thus.

Then bycause ther are counters in diuers lines, I shall begin with the highest, & take theym vp, setting for theym the multiplier so many times as I toke vp counters, that is twise. than wil the summe stand thus.



Then come I to the seconde lyne, and take vppe those 4 counters, settinge for theym th



the multiplier fower tymes, so wil the whole sum appeare thus.
So is the whole wages of 40 workmen for 28 daies (after 6 d eche day for a man) 6720 d, þ is 560 s, or 28 pounde.

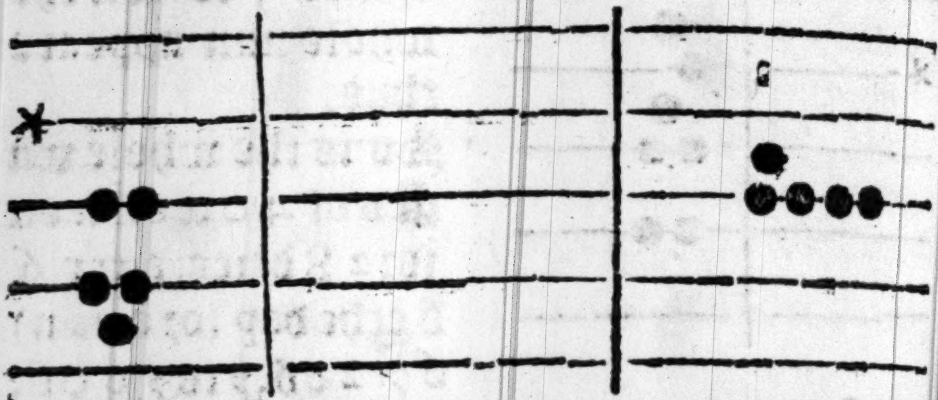
Mast. Nowe if you woulde proue Multiplication, the sureste waye is by Dyuysion: therfore will I ouerpasse it tyl I haue taught you the arte of Diuision, whiche you shall worke thus.

DIVISION.

FYrste set downe the diuisor, for feare of forgettinge, & than sette the numbre that shalbe diuided, at the right syde, so far from þ diuisor, that the quotiēt may be set betwene thē: as for exāple. Example of shepe

If 225 sheepe coste 45 li, what did euery sheepe coste? To knowe thys, I shoulde dyuide the whole summe, that is 45 li. by 225, but that can not be, therfore muste I fyrst reduce that 45 li, into a lesser denomination, as into shillinges: then I multiply 45 by 20 and it is 900: that summe shall I diuide by the numbre of sheepe, whiche is 225. these twoo num-

numbres therefore I set thus.



Then beginne I at the highest line of the dyuident, and seeke howe often I may haue the diuisour therin, and that maye I doo 4 tymes: then saye I, 4 tymes 2 are 8, why-che yf I take from 9, there resteth but 1, thus.

And because I founde the diuisonr 4 ti-mes in the diuidente, I haue sette as you see, 4 in the middle rome, whiche is the plac of the quotient: but nowe muste I take th reste of the diuisoure as often out of the re mayner, therefore come I to the second

lyn

lyne of the dyuisour, sayinge: 2 foure tymes make 8, take 8 from 10, and there resteth 2, thus.

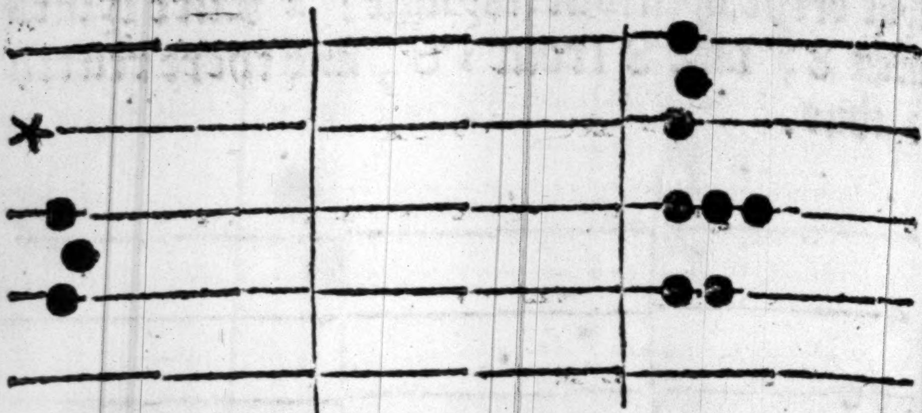
Then come I to the loweste numbre, whiche is 5, and multiplye it 4 tymes, so is yt 20, that take I from 20, and there remayneth nothinge: so that I see my quotiente to be 4, whiche are in valewe shillinges, for so was the divident: and therby I knowe that if 225 sheepe dyd coste 45 li, everye sheepe coste 4 s.

Scholar. This can I do, as you shall perceave by this example. If 160 souldiers do spende everye moneth 68 li, what spendeth eche man? Example of souldies wages.

Fyrste bycause I can not dyvide the 68, by 160, therefore I will turne the poundes into pennies by multiplication, so shall there be 16320 d.

Nowe muste I dyvide this summe by the numbre of souldiors, therefore I set theym in ordre, thus.

Then

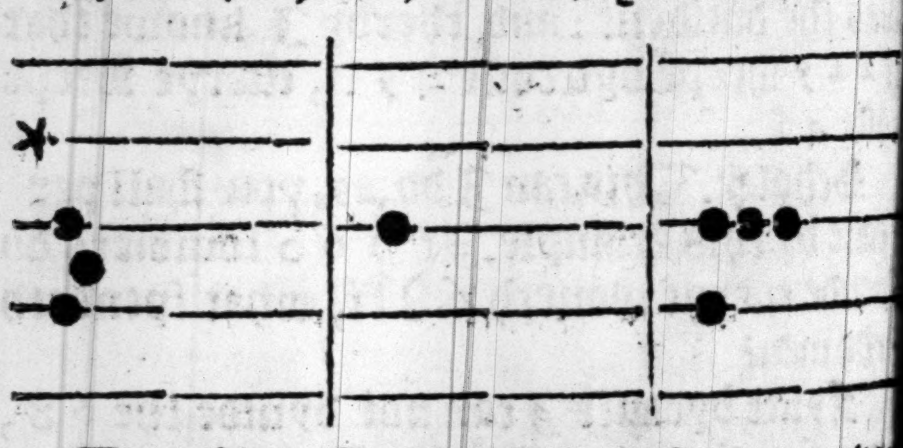


Than begyn I at the highest place of the diuident, seking my diuisour there, whiche I fynde ones, therfore sette I 1 in the nether lyne.

Ma. Note in the neither line of the whole summe, but in the neither lyne of that worke which is the thirde line.

Scholer. So standeth it with reason.

Master. Then thus do they stande.



Then seeke I againe the rest, how often I maye finde my diuisoure, and I see that in the 300 I myght finde 100 thre tymes, but then the 60 will not be so often found in 20 therfore I take 2 for my quotient; then tak

I 100 twise from 300, & ther resteth 100, out of which with the 20 (that maketh 120) I may take 60 also twise, and then standeth the numbres thus.

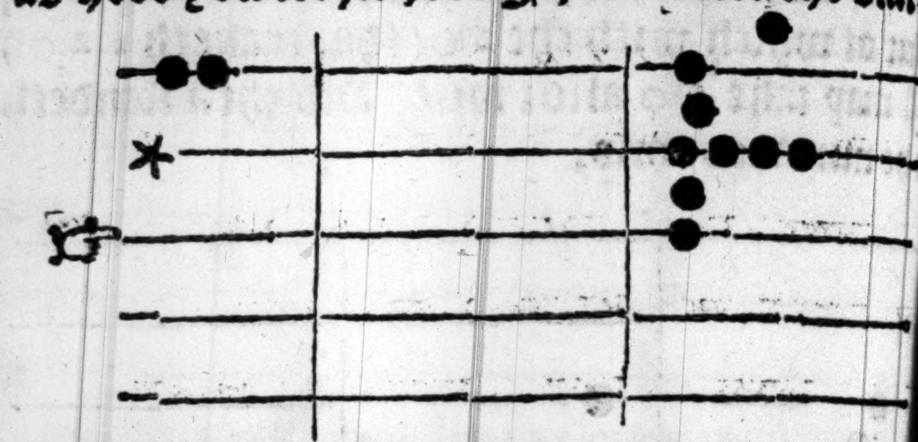
Wher I haue set the quotient 2 in the lowest lyne: So is euerye sowldiors portion 102 d: that is 8 s, 6 d.

Ma. But yet bicause you shall iustlye perceaue the reason of Diuision, it shall be good that you doo set your diuisour still againste those numbres from whiche you doo take it, as by this example I will declare.

If the purchase of 200 acres of grounde did coste 290 pounde, what dydde one acre coste? Example of purchas

Firste wyll I tourne the poundes into pennies, so will there be 69600 pence. Then in settinge downe these numbres, I shall doe thus. Firste sette the diuidente on the ryghte hande as it ought, and then the diuisor on the lefte hand against those numbres from which I intende to take him fyrste,

as here you see, where I haue sette the diui-

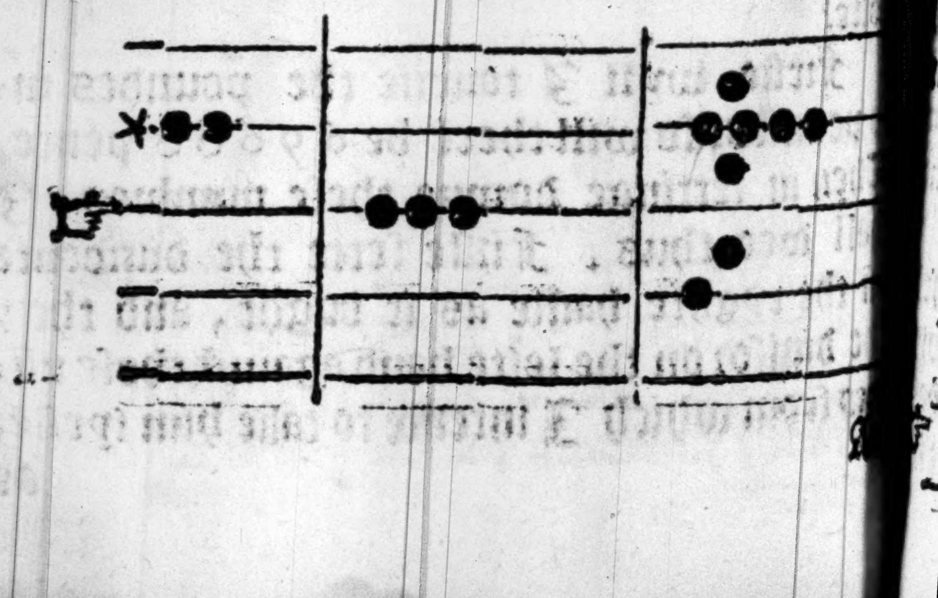


for two lines higher than is his owne place.

Sch. This is like the ordre of Diuision by the penne.

M. Truthe you say, and nowe muste I se the quotient of this worke in the thirde lyne, for that is the lyne of Unittes in respecte t the diuisour in this worke.

Then I seeke how often the diuisour ma be founde in the diuidente, and that I find 3 tymes, the set I 3 in the thirde line for th quotient, and take awaie that 60000 fro the diuident, and farther I doo set the diuisou one lyne lower, as you see heere.



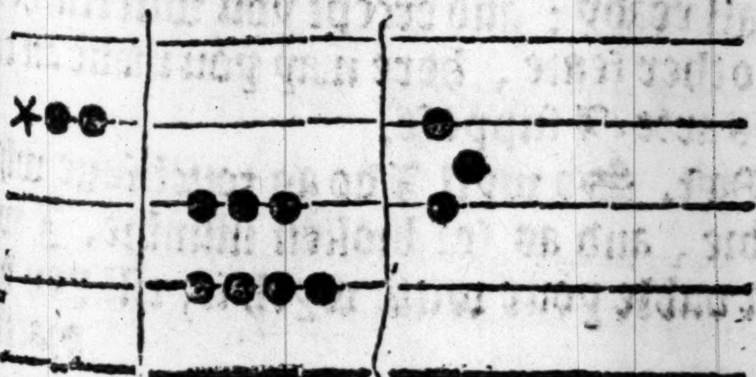
And then seke I howe often the diuisour will bee taken from the numbre against eyt, whiche will be 4 tymes, and 1 remaininge.

Schol. But what if it chaunce that when the diuisor is so remoued, it can not be ones taken out of the diuident againste it?

Master. Then muste the diuisoure be sette in an other line lower.

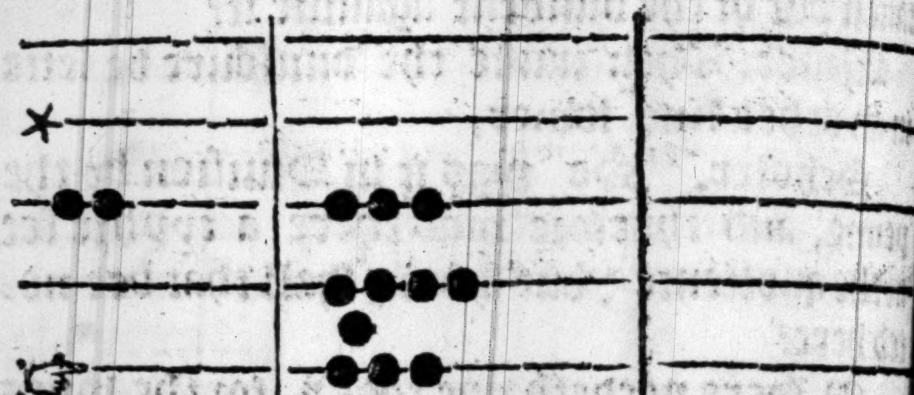
Scholer. So was it in Diuision by the penne, and therfore was there a cyphre set in the quotiente; but howe shall that bee noted here?

M. Here needeth noe token, for the lynes do represent the places: onelye looke that you set your quotient in that place whiche standeth for vnities in respect of the diuisoure. But nowe to returne to the exaumple. I find the dyuisoure 4 times in the diuident, & 1 remayning, for 4 tymes 2 make 8, whiche I take from 9, and there resteth 1, as this fygure folowinge sheweth: and in the middle space for the quotient I set 4 in the seconde line, whiche is in this worke the place of vnities.



Then

Then remoue I the diuisor to the nexte lo-
wer lyne, and seeke howe often I maye haue
it in the diuident, which I maye doo here 8
times iuste, and nothynge remayne, as in
thys fourme.



Wher you maye se e that the hole quotien
is 348 d, that is 29 s, wherby I knowe
so muche coste the purchase of one acre.

S. Now resteth the proues of Multipli-
cation, and also of Diuision.

M. Theire best proues are eche one by th
other: for Multiplication is proued by Di-
uision, and Diuision by Multiplication, a
in the worke by the penne you learned.

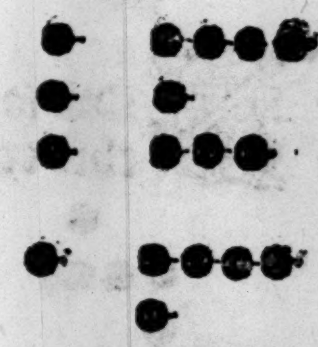
Scho. If that be all, you shall not neede t
repete againe that that was sufficientlye tau-
ghte all ready: and except you will teache m
anye other feate, here may you make an en
of this arte, I suppose.

Mast. So wyll I do as touchinge who
numbre, and as for broken numbre, I wy
not trouble your witte wyth it, till you ha
practis

practised this so well, that you bee full perfect, so that you neede not to doubte in anye pointe that I haue taught you, and than maye I boldelye instructe you in the arte of Fractions or Broken numbre: wherin I will also shewe you the reasons of all that you haue nowe learned. But yet before I make an ende, I will shewe you the ordre of common castinge, wherin are bothe pennies, shillings, and poundes, procedinge by no grounded reason, but onely by a receiued forme, and that dyuersely of dyuers men: for Marchantes vse one fourme, and Auditours an other.

MARCHAVNTES VSE.

But firste for marchauntes fourme, marke this example here, in which I haue expressed this sum 198 li, 19 s, 11 d. So þ you may se, that the lowest line serueth for pennies, the nexte aboue for shillinges, the thirde for poundes, and the fourthe for scores of poundes.



And farther you may see, that the space betwene pens and shillinges may receaue but one couter (as all other spaces likewaies do)

and that one standeth in that place for 6 d.

Likewayes betweene the shillynges and the poundes, one counter standeth for 10 s.

And betweene the poundes and 20 li, one counter standeth for 10 poundes.

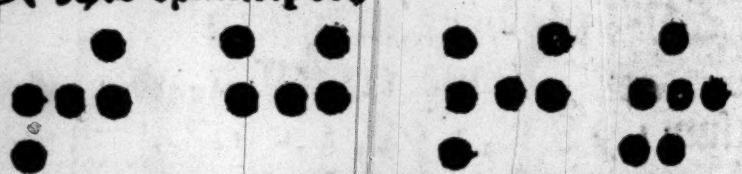
But beside those you maie see at the lefte side of shillinges, that one counter standeth alone, and betokeneth 5 s.

So against the poundes, that one counter standeth for 5 li. And againste the 20 poundes, the one counter standeth for 5 score poundes, that is 100 li. so that euery syde counter is 5 times so muche, as one of them against whiche he standeth.

AVDITOVRS ACCOMPT.

Auditours accompt.

NOw for the accompt of Auditors take this example.



Where I haue expressed the same sume 19 li. 19 s. 11 d.

But here you see the pennies stande towardes the righte hande, and the other encreasinge ordrely towarde the lefte hande.

Agayne you maye see that Auditours will make 2 lynes (yea and more) for pennyes, shillynges, and all other valewes, the

theyr summes extende thereto. Also you see, that they set one counter at the ryghte ende of eche rowe, which so set, there standeth for 5 of that roume: and on the lefte corner of the rowe it standeth for 10 of ye same rowe.

But nowe if you woulde adde other subtract after anye of bothe those sortes, if you marke the ordre of the other feate whiche I taughte you, you maye easylye doe the same heere wythout muche teachinge: for in Addition you muste fyrste set downe one summe, and to the same set the other ordrelye, & lykemanner if you haue manie: but in Subtraction you muste sette downe firste the greateste summe, and from yt muste you abate that other, euerye denomination from hys dewe place.

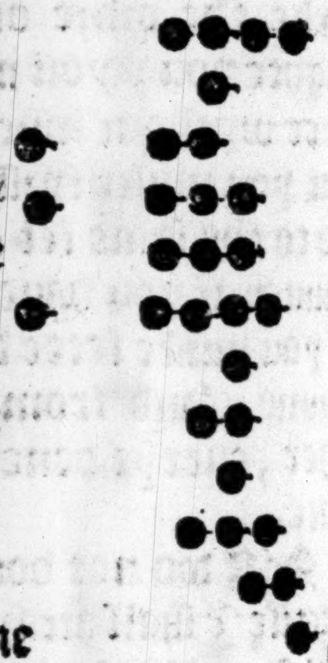
Sc. I doo not doubte, butte wyth a lyttle practise I shall attayne these both: but howe shall I multyplye and dyuide after these formes?

Mast. You can not duely do none of bothe by these sortes, therfore in suche case you muste resorte to youre other artes.

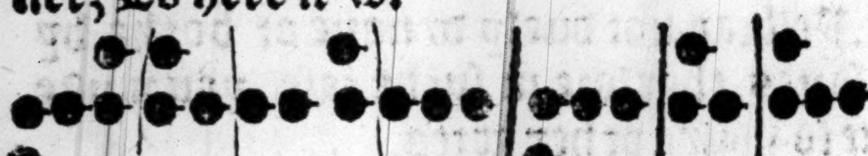
S. Syr, yet I see not by these sortes howe to expresse hundredes, yf they exceede one hundred, nother yet thousandes.

Mast. They that vse suche accomptes that it excede 100 in one summe, they sette not 5 at the lefte hande of the scores of poundes, but thei set al the hundredes in an other fac-

ther rowe, & 500 at the left hand ther of, & the thousandes they set in a farther rowe yet, & at the left side therof thei set ỷ 5000 & in ỷ space ouer, thei set ỷ 10000 & in a higher row 20000, whiche all I haue expressed in this example, which is 978,69li, 12s, 9d. ob. q̄. for I had not tolde you before, where nother how you shoulde set doune farthings, which (as you se here) must be set in a voyde space sidelinge beneth the pennies: for q̄, one counter, for ob, 2 cōters, for ob q̄, 3 cōters, & more ther can not be, for 4 farthinges do make 1 d, wiche muste be in his dewe place



And if you desire ỷ same sum after Auditors manner, Lo here it is.



But in this thing you shall take this for sufficient, and the rest you shall obserue as you maie se by the working of eche sorte: for the diuers wittes of men haue inuented dyuers and sundry waies, almoste vnnumerable.

Th

THE ART OF NVMBRING.
ON THE HANDE

Butte one feate I ſhall teache you, whiche not onely for the ſtraungenes and ſecretneſſe is muche pleaſant, but alſo for the good commoditie of it, ryghte worthye to be well marked. This feate hath ben vſed aboue 2000 years at the leaſt & yet was it neuer commonly knowen, eſpeciallye in Engliſhe it was neuer taughte yet, This is the arte of numbering on the hande, wyth diuers geſtures of ye fingers expreſſinge any ſum conceued in ye mind. And firſt to begin

If you wyll expreſſe any ſum vnder 100 you ſhall expreſſe it with youre lefte hande and from 100 vnto 1000, you ſhal expreſſe it with your ryghte hande, as here ordrelye by thys Table folowinge you may perceaue.

Here foloweth the Table of the arte of the hande.


1
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80
90
100
200
300
400
500
600
700
800
900
1000
2000
3000
4000
5000
6000
7000
8000
9000

1 In whiche(as you may see) 1 is expressed 1
by the lyttle fynger of the lefte hand, closelye
and harde crooked.

2 is declared by lyke bowinge of the wed- 2
dinge finger(whiche is the nexte to the little
fynger)together with the lyttle finger.

3 is signified by the middle fynger, bowed 3
in like manner with those other twoo.

4 is declared by the bowinge of the myd- 5
dle finger, and the ringe finger or weddinge
finger, with the other all stretched forthe.

5 is represented by the middle finger onely 5
bowed.

And 6 by the weddinge finger onelye croo- 6
ked; and this you maie marke in these a cer-
taine ordre. Butte nowe 7, 8, and 9, are ex-
pressed with the bowinge of the same fin-
gers, as are 1, 2, and 3, butte after an other
fourme.

For 7 is declared by the boowinge of the 7
little finger, as is 1, saue that for 1 the fin-
ger is clasped in, harde and rounde: but for
to expresse 7, you shall boowe the middle
iointe of the little finger onelye, and holde
the other iointes streight.

Scho. If you will giue me leaue to expresse
it after my rude manner, thus I vnderstande
youre meaninge: that one is expressed by cro
kinge in the little finger, like the heade of a
bishops bagle; and 7 is declared by the same
finger bowed like a gybbet.

Mast. So I perceiue, you vnderstande it.

8 Then to expresse 8, you shal bow after the same manner both the little finger & the ringe finger.

9 And if you bowe likewise with theim the myddle finger, than dothe it betoken 9.

10 Now to expresse 10, you shall bowe your forefynger rounde, and set the end of yt on the highest ioynte of the thombe.

20 And for to expresse 20, you must set your fingers straight, & the end of youre thombe to þ particion of the foremost & myddell finger.

30 30 is repreſẽted by þ ioining together of þ headdes of the formost finger & the thombe.

40 40 is declared by settinge of the thombe crossewaies on the formost finger.

50 50 is signified by right stretching forth of the fingers ioyntly, & applyinge of þ thumbes ende to the partition of the middle finger, and the ringe finger or wedding finger.

60 60, is fourmed by bending of the thomb croked, and crossing it with the forefinger.

70 70 is expressed by the bowing of þ foremost finger, and setting the ende of þ thombe betwene þ 2 formost or highest ioyntes of it.

80 80 is expressed by setting of the formost finger crossewaies on the thombe, so that 80 differeth thus frõ 40: that for 80, þ forefinger is sette crossewaies on the thombe, & for 40 þ thombe is set crosse ouer þ forefinger.

90 90 is signified by bendinge þ forefinger

an

and setting the ende of it in ẏ innermoste ioint of the thombe, that is euen at the fote of yt. And thus are al ẏ nūbres ended vnder 100.

Sc. In dede these be all the numbres from 1 to 10, & then al the tenthes within 100, but this teached me not howe to expresse 11, 12, 13, &c. 21, 22, 23. &c. and suche lyke. 11, 12

Master, You can little vnderstand, yf you can not do that without teachinge. What is 11? is it not 10 & 1? then expresse 10 as you were taught, and 1 also, and that is 11; and for 12 expresse 10 and 2: for 23 set 20 and 3: and so for 68, you muste make 60 and therto 8: and so of all other sortes. 13. 21 22, 23

But now yf you would represent 100, or ther anye numbre aboue it, you muste doe that with the right hande after this manner.

You must expresse 100 in the right hande with the little finger, so bowed as you dyd expresse 1 in the lefte hande. 100

And as you expressed 2 in the left hand, the same fashiō in ẏ right hand doth declare 200. 200

The fourme of 3 in the ryght hande stādeth for 300. 300

The fourme of 4, for 400. 400

Lykewayes the fourme of 5, for 500. 500

The forme of 6, for 600. And to bee shorter loke how you did expresse single vnities and tenthes in the left hande, so must you expresse vnities & tenthes of hundreds, in ẏ right hād. 600

Scholer. I vnderstande you thus: that if

900 I woulde represent 900, I muste so fourme
the fyngers of my right hande to expresse 9,
And as in my left hand I expressed 10, so in
my righte hande must I expresse 1000.

1000 And so the fourme of euerye tenthe in the
lefte hande, serueth to expresse like numbre
of thousandes, so the fourme of 40 standeth
for 4000.

4000 The fourme of 80 for, 8000.

8000 And the fourme of 90 (which is the grea-
test) for 9000, and aboue that I can not ex-
9000 presse any numbre.

Master. No not with one finger, howe be
yt, with diuers fyngers you may expresse
9999, & all at one time, and that lacketh but
1 of 10000. So that vnder tenne thousand
you may by your fingers expresse any summe.
And this shal suffice for Numeration on the
fingers. And as for Addition, Subtraction,
Multiplication, and Diuision (whiche yet
were neuer taughte by any man, as farre as
I do knowe) I will instructe you after the
treatise of fractions: and nowe for this time
far wel, & loke that you cease not to practise
that you haue learned.

S. Sir, with most harty minde I thank you,
both for your good learning and also youre
good counsell, which (God willing) I truste
for to folowe.

FINIS

THE SECONDE PART OF ARITHMETIKE touching Fractions, briefly sette forthe.

Scholar.

ALBE IT I PERceaue youre manifolde busynesse doothe so occuppye, or rather oppresse you, that yon canne not as yet completlie ende that treatyse of Fractions Arithmeticall, whiche you haue prepared, wherein not onely sundry workes of Geometrie, Musike, and Astronomye be largelye set foorth, but also diuers conclusions and naturall workes, touchinge myxtures of metals, and compositions of medicines, wyth other straunge exaumples, yet in the meane season I can not staie my earnest desire, butt importunelye craue of you some breefe preparation towarde the vse of Fractions, wherby at the leaste I maie be able to vnderstande the common woorkes of them, and the vulgare vse of those rules, which without them can not well be wrought.

Maister.

Master. If my leyser where as greate as my wyll is good, you should not nede to vse any importunate crauing for the attaininge of that thinge, wherby I maye be perswaded that I shall any waies profite the common wealthe, or healpe the honest studyes of any good membres in the same, wherfore while myne attendance wyll permitte me to walke and talke, I am well willinge to helpe you as I may.

What a Fractiō is. Therfore fyrste to begin with explication of this name Fraction, what take you yt to be?

Schol. Mary syr I thincke a Fractiō (as I haue harde it often named) to be a broken nūbre, that is to say, to bee no hole numbre, but a parte of a numbre.

Master. A Fraction in deede is a broken numbre, and so consequentlye the parte of an other numbre: but that muste bee vnderstande of suche an other numbre, as can not be deuided into any other partes then fractions, for althoughe I maye take the thirde parte of 60, or the fourth parte of it, and so of other partes diuersely, yet these partes be not proprely, nor oughte not to bee called Fractions, because they maye bee expressed by hole numbres for the thirde parte of it is 20: the fourth parte is 15: the twelfth parte is 5, and so forth of other partes, whiche all be hole numbres

Where

Wherfore proprely a Fraction expresseth the partes or part onely of an vnitie, that ys to say, that the numbre whiche is the whole or entire summe of any fraction, maye not be greater then one, & therfore it foloweth, that no one fraction alone can bee so greate, that it shall make 1, as by exaumples I wyll declare, as soone as I haue taughte you to knowe the forme howe a fraction is expressed or represented in writinge.

NVMERATION

But fyrst to begin with ye expressinge of a fraction, which is the Numeration of it, you muste vnderstande, yt a fraction is represented by 2 numbres set one ouer the other, & a line drawen betweene thẽ, as thus $\frac{1}{3}\ \frac{3}{4}\ \frac{2}{5}\ \frac{10}{17}$ whiche foure fractions you must pronounce thus: $\frac{1}{3}$, one third parte: $\frac{3}{4}$ thre quarters: $\frac{2}{5}$, two fifte partes: $\frac{10}{17}$, ten seuentene partes.

Scho. I vnderstand the forme of theire expression and pronuntiation, but their meaning or valuation semeth more obscure: yet I thinke that by the two fyrste fractions I vnderstande the valuacion of the two latter fractions, and so consequently of other.

Ma. Valewe them than, that I may perceaue

ceaue your taking of them.

Sch. $\frac{2}{5}$ betokeneth two fyfte partes, that is to saye, if one bee diuided into 5 partes, that fraction dothe expresse ij. of them: so $\frac{10}{17}$ dothe signifie, that if one be diuided into xvij partes, I muste take x. of them. And this I gather of the ij. fyrste axaumples, for $\frac{1}{3}$, that is one thyrde parte, doth easilye declare, that if any one thynge bee deuided into iij. partes, I muste take but one of them, so $\frac{3}{4}$, that is iij. quarters, doth declare that one beinge diuided into iiij. quarters, I muste take for this fraction iij. of those quarters.

If there bee no more difficultie in theyre Numeration, then I praye you go forwarde to their Addition and Subtraction, and so to the other kindes of workes: for I vnderstande that the same kindes of workes bee in Fractions, that be in whole numbres.

Ma. There are the same kynde of workes in bothe, albeit the ordre of them is diuers, as I wil anone declare. But yet more in numeration before we leaue it: You muste vnderstande, that those two numbres whiche expresse a fraction, haue seuerall names, the ottermoste, whiche is aboue the line, is called the Numerator, and thother beneth the line, is called the Denominator.

Sc. And what is the reason of their diuers names? for in mine opinion both the numerators, seinge bothe thei do expresse the numeratio

ration of the fraction.

M. You are deceiued: for one onely (which is the ouermoste) dothe expresse the numeration, and the Denominator dothe declare the numbre of partes into whiche the Unitie ys diuided. as in this example, when I say: Diuide a pounde weighte of golde betwene iiij. men, so that the firste man shall haue $\frac{2}{15}$, the seconde $\frac{3}{15}$, the third $\frac{4}{15}$ and the fourthe $\frac{6}{15}$. Nowe doe you perceaue that by the denominator (which is one in all foure fractions) it is intended, that the pound waight should be diuided into so manye partes, I meane 15, and by the iiij. seuerall numeratours is lymited the diuers portion that eche manne shall haue, that is, yͭ when the whole is parted into 15, the firste mā shall haue 2 of those 15 partes: the seconde man 3 of them: the thirde man 4: and the fourth man 6: And so may you see the seueral offices (as it wer) of those two numbres, I meane of the Numerator and the Denominator.

And hereby you perceaue, that a man can haue no more partes of any thinge, then yt was diuided into: nother yet aptly so many. So that it were vnaptly saide: You shall haue $\frac{15}{15}$, that is xv. fiftene partes of any thinge, seeinge it were better saide: You shall haue whole thinge.

S. So dothe it appere reasonablye, for the laboure is vayne to diuide anye thinge

and

and then to applye the Diuision to no vse.
And muche lesser reasonable wer it to say $\frac{16}{15}$.
for if the whole bee dyuided into 25 partes
onelye, it is not possible to take 16 of them,
that is to say, more then altogether.

M. This is true, touchinge the propre and apte vse of the name of a fraction: yet improprely and after a vulgare acceptation (for easynes in work) both those fourmes bee called Fractions, bicause they bee written lyke Fractions, although they bee none in deede, for $\frac{5}{5}$, and generallye all suche other where the Numerator and Denominator bee equall, are not fractions, but the whole thing with all his partes. And so $\frac{16}{12}$, is not to bee called a fraction, but a mixte nūbre of a whole numbre and a fraction, for it is as much as $1\frac{4}{12}$, that is one whole one, and 4 twelue partes, as shall bee declared in Reduction. Therefore they do abuse the names that call them fractions, where the Numerator is other equall or greater then the Denominator.

Sch. But is there any needeful cause why they shoulde so abuse the name?

Ma. There is cause why they shall sometimes for easinesse in worke write some numbres after that sorte lyke fractions, but they neded not to call them fractions, but as they be Whole numbres, or Mixt numbres (that is whole numbres with fractions) expressed lyke fractions.

Nowe

Nowe muste you vnderstande, that as no fraction proprely can be greater then 1, so in smalnesse vnder one the nature of fractions doth extende infinitely: as the nature of whole numbres is to increase aboue one infinitelye, so that not onely one may bee diuided into infinite fractions or partes, but also euerye fraction may be diuided into infinite fractions or partes, whiche commonly be called Fractions of fractions, and they bee expressed diuerslye: As for exaumple: $\frac{3}{4}\frac{2}{3}\frac{1}{2}$, that is three quarters of two thirde partes of one halfe parte. Wherbye is signified, that if one be diuided into two halues, and the one halfe into three partes, and two of those three partes be diuided ioyntlye into foure quarters, this fraction dothe represent thre of those quarters.

S. I pray you let me proue by an example in common money, whether I do rightelye vnderstande you or no. One crowne whyche I take for an vnitie, dothe containe 60 pennies, therfore the halfe of it is 30 pence $\frac{2}{10}$ of that halfe is 20 pence, wherof 3 is 15 pence: so than 15 pence is $\frac{3}{7}\frac{2}{3}\frac{1}{2}$ of a croune. And so 3 pence is $\frac{3}{4}\frac{2}{3}\frac{1}{2}$ of a shillinge.

M. You perceaue this well ynough, but how happened that you founde no doubte in the forme of writinge these fractions, seyng the two latter fractions haue no line betwen their numbers, as the firste hath?

S. Bicause I had forgotten (as scholers ofttimes do) that that was tolde me before: but I pray you expresse the reason therof.

M. This forme is but voluntary, and therfore hathe none other reason, then the wil of the diuisor, whiche forme manye doe folowe. Some other doe make lynes betwene euerye fraction, and adde wordes of distinction after this sorte, $\frac{3}{4}$ of $\frac{5}{1}$ of $\frac{1}{2}$, whiche forme is good also.

Some other expresse them thus in slope forme, to distincte them from seuerall fractions of one whole number. for if they were sette in one right line thus, $\frac{3}{4}\,\frac{2}{3}\,\frac{1}{2}$, than oughte it to bee pronounced, thre quarters, and two thyrde partes and an half, whiche maketh almoste two whole vnities, lacking but one xij parte. And so is it nothinge agreable with the other Fraction of fractions. wherefore it is a great ouersight in certaine learned men, which doe expresse them so confusedly with suche seuerall fractions, that a man can not knowe the one from the other.

$\frac{3}{4}$
$\frac{2}{3}$
$\frac{1}{2}$

Therfore summe menne (as Stifelius) doe expresse without a lyne, noumbers of proportion, beynge applyed to Addition or subtraction, bicause they muste be taken as twoo, where the lyne in fractions maketh theym to bee taken for one. for of the Numeratour and Denominatour is made one

numb-

numbre.

S. Than I perceaue ther be thre seuerall varieties in fractions: Firste whan one only fraction is sette for one numbre, as $\frac{4}{5}$, that is foure fifte partes. The seconde, is whē ther be set two or more seuerall fractions of one number, as $\frac{4\ 2}{9\ 5}$, that is iiij nynthe partes & two fift parts. The thirde sort is fractiōs of fractions, as $\frac{4\ 2}{5\ 6}$, that is iiij nynthe partes of two fift partes.

M. You haue said well, if you vnderstand well youre owne wordes

S. If it shall please you I wyll by an example in the partes of an olde Englishe Angell expresse my meaning.

M. Let me here you.

S. The olde english Angell did contayne vij. s. vi d. that is 90 d. Nowe $\frac{4}{5}$ of it, is 72 d and of the same 90 d, if I take $\frac{4\ 2}{9\ 5}$, that is foure nynthe partes, and two fift partes. $\frac{4}{9}$ is, 40, and $\frac{2}{5}$ is 36, whiche bothe make 76 but yf I take $\frac{4\ 2}{9\ 5}$, that is foure nynthe partes of two fifte partes, seynge $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36, will yelde but 16, for $\frac{1}{9}$ of 36 is but 4, and that taken foure tymes, maketh 16.

M This is plainly expressed and truly, and herby (I doubt not) but you do perceaue, that as greate a difference as is betweene 16 and 76, so much difference is betwene these two fractions $\frac{4\ 2}{9\ 5}$, and $\frac{4\ 2}{9\ 5}$

The wordes of Fractions

And nowe that you vnderstande these varieties, I will procede to the reste of the workes: fyrste admonishinge you, that there is an other ordre to be folowed in fractions then there was in whole numbres. for in whole numbres this was the ordre, Numeration, Addition, Subtraction, Multiplycation, Diuision and Reduction: but in fractions (to folowe the same aptnesse in procedinge from the easyest woorkes to the harder) we muste vse this ordre of the workes, Numeration, Multiplication, Diuisiō, Reduction, Addition, and Subtraction.

Sc. That Multiplication and Diuision shoulde go togither, and Subtraction to folow Addition, naturall ordre doth persuade: but why multiplycation shoulde be firste in ordre here nexte Numeration, and Reduction in ȳ myddle, I desyre to vnderstande ȳ reason.

M. As in the arte of whole numbres ordre woulde resonablye begynne with the easiest, and so goe forwarde by degrees to the hardest, euen so reason teacheth in Fractions the lyke ordre. And considering that Addition or Subtraction of Fractions can very seldome be wrought without Multiplycation and Reductyon: and contrary wayes Multiplycation and Reduction maye be wroughte without this forme of Addition or Subtraction, therfore was it ordrely required, that Multiplication and Reduction

shoulde

shoulde goe before Addition and Subtraction. And the same reason serueth for the placinge of Multiplication before Reduction.

Sc. Then if Multiplication be the easiest, I praye you declare the forme of it, fyrste by rule, and then by example.

Maister. Your request is good.

MVLTIPLICATION.

Therfore whē any two fractiōs be proponed to bee multiplied together, the Numerator of ẏ one must be multiplied by thee Numerator of the other: & the sum ẏ amounteth therof, must be set for a newe numerator: likewaies ẏ Denominator of the one must bee multiplied by ẏ denominator of the other, & that that amounteth shalbe set for the common denominator, & this newe third Fraction expresseth the multiplication of the two first fractions proponed: whereof take this example $\frac{3}{5}$ multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$.

S. I perceaue then that 3 beynge the numeratour of the firste fraction, is multiplied by 5, beinge the numeratour of the seconde Fraction, wherof amounteth 15, the Numerator of the thirde fraction. And so lyke waies 5 beinge denominator of the first Fraction, is multiplyed by 12 the Denominator of the second fraction, wherof amounteth 60

the newe denominatour, so that I perceaue how the work is done. but I doe not perceiue how $\frac{15}{60}$ is greater then $\frac{3}{5}$. For if I shall vse my former manner of examination by the partes of some coyne, I se that $\frac{3}{5}$ of a crowne is 36 d. and $\frac{5}{12}$ of a crowne, is 25 d, wherof the one multiplied by the other, dothe make 900 d, which is 15 crownes. but by your multiplication there amounteth $\frac{15}{60}$, whiche is but 15 d, and that is much lesser then any of both the first fractions.

M. What difference is betwene multiplycation in whole numbres, and multiplication in broken numbers, that in whole numbres the sum that amounteth, is greater then bothe the other wherof it came: but in fractitions it is contrary wayes: for the sum that amounteth, is lesser then anye of ye other two fractions, wherof it came.

S. I desire muche to vnderstande the reason therof.

M. Althoughe I purposed to reserue the reasons of works Arithmeticall, for the perfect boke of Arithmetik, yet I will shew you this, bicause of the straungenes of the worke.

You see in whole numbres, that of two numbres beinge multiplied together, is made the thirde numbre: whiche thirde numbre dothe beare the same proportion to the number multiplyed, that the multiplyer dothe beare to an vnitie. And so in fractions the

the thirde noumbre whiche amounteth of multiplication, beareth the same proportion to eche of the two firste Fractions, that the other of those two Fractions doth beare to an unitie.

Scholar. Sir I understande youre wordes thus: Whan 40 is multiplied by 12, there dothe amounte 480, whiche 480 dothe containe 40 so manye tymes in it, as 12 dothe containe unities: that is to say, twelue tymes. And so it appeareth that 480 dothe contayne twelue so many times also, as 40 dothe containe unities, that is 40 tymes. But nowe I see not howe the thirde numbre in this example of Fractions, can contayne anye of the twoo former (as it happened in whole numbres) seyng it is lesser then eyther of them.

Master. No meruaile if you can not se that thynge whiche is not possible to bee seene of anye man, howe the thirde noumbre in multiplication of Fractions shoulde bee greater than any of the two former Fractions, but yet this may you see (whiche I saide) that the thyrde numbre in fractions so multiplied, dothe beare the same proportion to anye of the two former Fractions, that the other of those ij Fractions dothe beare to an unitie, as in your example $\frac{3}{5}$ beinge multiplied by $\frac{5}{12}$, dothe make $\frac{15}{60}$. Nowe saye I, that $\frac{15}{60}$ dothe beare the same proportion to $\frac{3}{5}$, that $\frac{5}{12}$

 dothe

dothe beare to an unitie, as you may in youre owne forme of examination by coyne trye it. For in an olde Angell are 180 halfe pence, which I sette for the intire unitie, whose partes accordinge to the Fractions afore-sayde, are these, for $\frac{15}{60}$ set 45 ob. for $\frac{3}{5}$, take 108 ob. and for $\frac{5}{12}$ put 75 ob. Nowe dothe 45 beare the same proportion to 108, that 75 doth beare to 180. for 45 is $\frac{5}{12}$ of 108, & so is 75 also $\frac{5}{12}$ of 180.

But these reasons maye bee better reserued till an other time, when the knowledge of proportions in due ordre shall bee taughte. Yet in the meane season I will shewe you howe it commeth to passe, that in fractions þ thirde summe must nedes bee lesser then anye of the other two.

Considre thus, that when a Fraction is proponed as in the former example $\frac{3}{5}$, if it bee multiplied by more then one, it will make more then one intiere numbre. As if I multiplye this $\frac{3}{5}$ by 5, that is to saye, if I take yt fiue times, yt wyll make three entiere unities: exaumple in a crowne, $\frac{3}{5}$ of it maketh 3 ß, whiche if I take fyue tymes, it will amounte to 15 shillinges, that is three entiere crownes. so if I take the same $\frac{3}{5}$ but twise, it will yelde 6 ß. that is one entier crowne and $\frac{1}{5}$. Nowe if I take it but ones, it can not be more than it was before, that is 3 ß. And if I take it lesse then ones,

can not be so much as it was before. Then seynge that a Fraction is lesse then one, if I multiply a Fraction by an other Fraction, it foloweth, that I doe take that firste fraction lesse then ones, and therfore the summe that amounteth muste needes be lesse thē the firste fraction.

Scholar. Sir I thanke you muche for this reason: And I truste, I doe perceaue the thinge, as by example of this same fraction $\frac{3}{5}$, I will expresse. If I take $\frac{3}{5}$ of a crowne ones, that is to say, if I multiplye $\frac{3}{5}$ by 1, it will bee as it was before, but 3 ß. so if I doe multiplye it by $\frac{1}{2}$, that is, yf I take yt but halfe one time, than will yt bee but half so much: likewaies, yf I multiplye it by $\frac{1}{3}$, that is, if I take but the third part of ones, it will yelde but 12 pence, that is the thirde parte of the firste Fraction.

And so to make an ende. If I take it but the twelfthe parte of ones, that is, if I doe multiplye it by $\frac{1}{12}$, yf yt will yelde but the twelfthe part of the fyrste fraction, which is but 3 pence. And so it foloweth. that if $\frac{1}{12}$ make 3 pence, than $\frac{5}{12}$ muste needes make fyue times so much, that is 15 pence. which was the summe that hathe giuen the occasion of all this doubte.

Mayster. Then I perceaue you haue sufficiente vnderstandinge in this sorte of Multiplication for this tyme, wherefore

I wyll omitte that I mighte ſaye more of Multiplication, tyll wee come to reduction, and wyll paſſe to the other workes, and firſte to Dyuyſion, whoſe place foloweth Multyplycation, bothe by naturall order, and alſo in eaſineſſe of worke.

DIVISION.

When ſo euer two Fractions be proponed, that the one ſhoulde be diuided by the other, I muſt ſet downe firſt the fractiō that ſhall be diuided (whiche is called the Diuidend) & then after yt the other, which is ỹ Diuiſour. Then ſhall I multiply the numerator of the diuidend by the denominator of the diuiſor, and ỹ whiche amounteth, I muſte put for a newe numerator: Again, I ſhall multiply the denominator of the diuidend, by the numerator of the diuiſour, and the numbre that amounteth thereof, I muſte put for the newe denominatoure. And this third fraction is the quotient of the ſayd diuyſion.

Scholer. This ſemeth eaſie in forme, as by example, thus: If I woulde diuide $\frac{5}{8}$ by $\frac{2}{6}$, fir[st] I muſt multiply 5 (beinge ỹ nummeratour of the diuidend) by 6, which is the denominat[or] of the diuiſour, and therof riſeth 30: then I

mu

multiplye 8 (beinge the denominatoure of the diuidende) by 2, beinge numeratoure in the diuysor, and so ryseth 16. the whiche I must make in a thirde fraction, thus, $\frac{30}{16}$.

Master. Me seemeth you are quicker in vnderstandinge nowe, then you were when I taughte you the arte of hole numbres: but that is no maruaile, for the more knowledge that anye man getteth, the readyer shall hee fynde his witte, and quicker in vnderstandinge. but yet of ij. thinges I will admonishe you, whiche you mighte haue obserued heere, for ease of worke, and lightnesse of vnderstandyng, the nature of the Quotient.

When soeuer you doe diuide one fraction by an other, other they bee bothe equall togyther, other elles the one is greater then the other: yf they bee equall, there Quotiente shall bee suche, that the Numeratour and the Denominatour of it shalbee equall allso. And if the two firste fractions be vnequal, their quotient shall declare the same. by the vnequalitie of the numeratour and denominator, as in these examples folowinge shall appeare.

Fyrste of equall Fractions: $\frac{4}{3}$ and $\frac{12}{27}$, bee equall together: and if the one be diuyded by the other, the quotiente wyll bee $\frac{108}{108}$, as you may perceaue by that rule aforesaide.

Nowe in vnequall fractions, as $\frac{4}{9}$ and $\frac{7}{18}$ the quotient will be $\frac{42}{27}$: wher the numerator

is

is greater then the denominator.

Sc. I see it is so, but I see not the reason why it shoulde be so.

M. The reason is this. When any fractiō is dyuided by an other, the quotiente declareth what proportion the diuidente beareth to the diuisour. So $\frac{1}{2}$ dyuided by $\frac{1}{4}$ maketh 2, whiche muste bee sounded not two, butte twise: declaring that $\frac{1}{4}$ is contained twise in $\frac{1}{2}$.

Note. And note this, that the Numerator in the Quotient representeth the Diuidende, and the denominator representeth the diuisor. And this is alwaies true, whether the greater fractions bee dyuided by the lesser, or the lesser by the greater. But this proportion will not bee exactly knowen, till you haue learned the art of Proportions: not withstandinge somewhat of it will I declare in the nexte rule of Reduction. But nowe for the easy remembrance of the quotiente in diuisiō as sone as you haue set downe your ij. fractions, the one againste the other, then make a line for the quotiente: and as soone as you haue multiplied the numerator of the diuidend, by the denominator of the deuisour, set the numbre that amountethe ouer the sayd lyne, and then multiplye the other two numbres, and set their total vnder the same line.

S. I perceaue you wold not haue me trust to memory til I were better expert, leaste ofttentime

tentimes I happen by mysse remembraunce to be abused. This example I take for that declaration.

If I woulde diuide $\frac{2}{3}$ by $\frac{3}{4}$, I muste set the nũbers one againste the other, (as here doth appeare) & then make an other line for the quotiẽt in some good distaunce. where I may sette the numbres of the quotient, as sone as anye of them is multiplied: So then as soone as I haue multipled 2 by 4, which maketh 8, I shall set that 8 ouer that line, thus, $\overline{8}$, and then multiplye 3 by 3, whiche yeldeth 9 and that 9 muste I set vnder the same lyne, and then will the whole quotiente appeare thus, $\frac{8}{9}$. Wherby appeareth (as I remembre, your wordes) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$, as 8 is to 9. but how may I perceaue that?

$$\frac{2}{3} \qquad \frac{3}{4}$$

M. Although you shall better percaue it by the rule of Reduction, yet this example may be declared in common coyne, as in a common shilling of xij. pens, of whiche $\frac{2}{3}$ maketh 8 d, and $\frac{3}{4}$ doth make 9 pens, and so you may easilye see that theire proportions do agree. And if you had taken this exam-ple before, when you toke the example of $\frac{5}{2}$ and $\frac{2}{6}$, youre quotient wolde appear (as thys doth) more easyer to vnderstande, where as that quotient being $\frac{30}{16}$, is not an easye pro-portion for you to perceaue, beyng yet little acquainted with proportions: whereof to

gyue

gyue you some taste, I wyll entre to the rule of Reduction, in whiche also I wyll declare other workes, bothe of Multiplication, and also of Diuision, whiche nowe I muste for a time omit, as things that do neede the helpe of Reduction.

REDVCTION.

Fyue varieties of Reduction

THerfore wyll I nowe declare the diuersities of Reductiō of Fractions, whiche commonly hathe fiue varieties,

1. First, when ther be sundry Fractions, of one intier vnity, they muste be reduced to one Denomination, & also into one Fraction.

2. Secondarily, when there bee proponed fractions of Fractiōs, they muste be reduced likewise into one fraction, for other waies they cānot be brought into one denominatiō.

3. Thirdly, when an Improper Fractiō is proponed, that is to saye, a fraction in forme whiche in dede is greater then an vnitie, it muste be reduced into apte forme, expressing the vnitie or vnities of it, and the proper fraction distinctly. And some times also it shall be needefull to conuerte suche a mixt nū‑bre of vnities with fractions into the forme of a fractiō, that is into an Improper fraction, whiche 2 formes I esteme but as one because

bicause they worke on one kinde of numbre.
4. Forthly, there appereth somtimes fractions to be written in great numbres, whyche might be wrytten in lesser numbres, therfore is there a meane to reduce suche greate numbres into theyr smalest tearmes

5. Fyftly, when any fraction betokeneth the partes of a whole thinge, whiche hath by cōmon partition certayne partes, but none of lyke denomination with that fraction, then may you reduce the sayde fraction into another, whose denomination shall expresse the cōmō partes of that whole thyng.

Sco. This distincion in doctrine, delyteth me muche, but more with hope then presente frute, for as yet I do not understande scarcelye the varieties, and muche lesse the practise and vse of their workes.

Ma. Reduction is an ordrely alteration of numbres, out of one form into an other. whiche is neuer done ordrely but for somme needefull vse, as in euerye of the said 5 seuerall varieties I will distinctly declare.

Fyrste therfore, when two or more seueral fractions of any unitie bee proponed, as for example, $\frac{6}{7}$ $\frac{1}{3}$ and $\frac{4}{8}$: bycause it is harde to tell what portion of the intiere numbre these two fractions doo expresse, therefore was reduction diuised, to bee a meane whereby these seuerall fractions myghte bee broughte into one denomination and fraction.

The firste sorte of reduction

And

Howe to reduce Fracttons of diuers Denominations into one Denomination.

And in these fractions this is the arte for bringing them to one denomination.
Multiply firste the denominatours togither, and the totall therof you shall sette twise downe vnder two seuerall lynes for twoo newe denominatours, or rather for one common denominatour: Then multiply the numeratour of the firste fraction, by the denominatour of the seconde, and sette the totall therof for the numerator ouer the fyrste line. Lykewise multiplye the numerator of the seconde fraction by the denominatour of the fyrste, and sette that totall ouer the seconde lyne for the numeratour of that fraction, and so are those two fyrste fractions of seuerall denominations brought to one denominatiō.

Scho. If I vnderstande you, as I thinke I doe, my example shall declare the same. The fractions whiche you proponed were these $\frac{3}{16}$ & $\frac{4}{6}$, whose denominatours (beyng 16 and 6) I multiplye togather, and then amounteth 96, whiche I set vnder ij. lynes thus, $\overline{96}\ \ \overline{96}$. Then I multiply the numerator of the firste fraction by the denominatour of the seconde, saying: 3 into 6, maketh 18, that set I ouer the firste lyne for new numerator, and it will be thus. $\frac{18}{96}$

Likewaies I multiply the numeratour of the seconde Fraction by the denominatour of ỹ fyrste, saing: 4 tymes 16 maketh 64, that I set for the seconde numer

for, and the fraction $\frac{64}{96}$
wyll appeare thus:
so that both Fractions broughte to $\frac{18}{96}$
one denomination, must stand thus.
and $\frac{64}{96}$

Maister. You haue done well.

Sch. I beseecke you, let me examine it after my accustomed tourne, by common partes of coyne.

Maister. So to.

Scho. A newe Angell accompted at eight shillinges, containeth 96 pens, whereof $\frac{1}{16}$ that is the xvj part, is syx pence, and $\frac{3}{16}$ is 18 pence, that is $\frac{18}{96}$. Againe $\frac{4}{6}$ of the same Angell, is 16 pens, so that $\frac{4}{6}$ maketh 64 d. that is $\frac{64}{96}$. And so I finde the summes to agree with thother before.

M. So haue you nowe the arte to bringe suche two Fractions into one denomination. And if there be more then ij, then muste you multiply all the denominators together, and set the total therof so manye tymes down as ther be fractions, and the to get for eche one newe numeratour. Multiplye the numerator of the firste, by ye denominators of the second, and the totall therof multiplye by the denominator of the third, and so forth if ther be more. Likewise multiply e the numerator of the second, by the denominator of the firste, and the totall therof by the denominator of

the thyrd. And in the same sorte multiply the numerator of the thirde, into denominator of the firste: and the total therof into ẏ denomi- nator of the secōd, & so forth if ther wer moe. So these 3 fractions $\frac{2}{5}$ $\frac{3}{4}$ $\frac{2}{3}$ dothe make by re- duction these other 3 fractions of one deno- mination $\frac{24}{60}$ $\frac{35}{90}$ $\frac{46}{60}$. All which you may bring into one fraction by addinge the numerators together, & putting that totall for the cōmō numerator, reseruinge still that same common denominator. And those 3 fractions make one improper fraction, thus, $\frac{109}{60}$.

Scholer. Al this I perceiue, and also that this laste fraction is more then an vnitie, and therfore you did cal it an Improper fraction.

M. There be certaine other formes of wor- king in this Reduction, whiche I will brief- lye touche also, to gyue you an occasion of ex- ercise your wit therin.

The firste variety of this re- duction.

The firste varietie is this. When you haue made and written downe youre common de- nominator (as I haue taught before) then to get a numerator for the firste, do thus. Di- uide the common denominator by the deno- minator of the first fraction, and the quotient multiplied by the numerator of the same yeldeth a newe numerator for the firste new fraction. So likewise do with the second and the third, & with al ẏ residue, if ther be more.

Sch. That wil I proue in your laste exam- ple of these 3 fractions $\frac{2}{5}$ $\frac{3}{4}$ $\frac{2}{3}$. When the

de

denominatoures bee multiplied, they make 60, for 5 into 4 maketh 20, & 20 by 3 yeldeth 60, that I sette downe 3 times, thus. $\overline{60}$ $\overline{60}$ $\overline{60}$. then to haue a numeratoꝛ for the firste, I must diuide 60 by 5 (the denominatoꝛ of the first) & the quotient is 12, which I must multiply by 2 (the numeratour of the first) and that maketh 24, and so haue I for the firste fraction $\frac{24}{60}$.

Likewise for the seconde fraction I diuide 60 by 4, & there commeth 15, which I multiply by 3, and so haue I 45, and the seconde fraction $\frac{45}{60}$. Then for the thirde in like sort will come. $\frac{40}{60}$.

The seco[nd] varietie.

Ma. An other way is this. If it happen so ŷ the lesser denominatour can by anye multiplication make the greater, then note the multiplier, and by it multiplye the numeratour ouer that lesser Denominatour, and for the lesser Denominatoure putte the greater, as thus in these twoo Fractions, $\frac{[illegible]}{12}$ $\frac{2}{3}$, three beinge the lesser denominatoure multiplyed by 4 will make 12, which is the greater denominatour: therfore by the same 4 I doe multiplye 2, whiche is numeratoure ouer 3, and that maketh 8: vnder whiche I put 12 beinge the greater denominatoꝛ, whiche is also made by multiplication of 4 into 3, and so haue I these twoo fractions $\frac{1}{12}$: [illegible] thus shortelye reduced without alteringe the one fraction.

The thirde varietie.

Scholer. This I vnderstande.

Maister. Then marke this thirde way: Yf the denominators do not happen so, that one by multiplication may make the other, then looke whether they both may be partes of anye other one numbre, as in $\frac{5}{12}$ and $\frac{7}{18}$, although the lesser taken but twyse, bee to great to make 18, yet they both may be partes vnto 36: therfore loke how many times 12 is in 36, and that quotient beinge multiplied by the numeratour ouer 12, the total shalbe put in steede of the numerator ouer 12 and for 12 put 36, thus, $\frac{15}{36}$. So likewaies looke how often is 18 in 36, and bicause it is twyse, therfore by 2 multiplye 7, whiche is ouer 18, and it will be 14, set that for the numerator, and in steede of 18 put 36, & then shal youre Fractions reduced stande thus. $\frac{15}{36}$ $\frac{14}{36}$ in steede of $\frac{5}{12}$ and $\frac{7}{18}$.

And if you will proue whether you haue wrought wel or no, that may be proued by Reduction of them againe to theyr former denominacions, which arte shall bee taught in the fourth kynd of Reduction, where greater termes of fractions be reduced into smaller in number, but no smaller in proportion. And if in suche Reduction the same termes or numbers come againe that were before than is the worke good, els not.

Scholer. Sir, I heare youre wordes, but I doe not vnderstande many of them, whi

it may please you to declare.

Maister. With a good will, whan conuenient place serued. but that must be in the sayde ..ng kinde of Reduction. In the meane season I will declare ẏ seconde forme of Reduction, which teacheth howe to reduce fractions of fractions into one fraction, and so to one Denomination.

When fractions of fractions be proponed you shall multiplye the numeratours of eche into other, and sette the totall for the new numeratour, than multiplye all the denominatours likewaies, and take their totall for the new denominatour, and so are they spedely reduced.

Reductiō fractions of fractions into one fraction and Denomination.

Scholer. If that bee all, then I vnderstād it al redy, as by this example I will declare. These bee the Fractions, $\frac{3}{4}$ $\frac{2}{3}$ $\frac{6}{7}$ $\frac{7}{9}$, whiche I woulde reduce to one denomination. Therfore beginne I with the numeratours, and multiply them altogether, sayenge: 3 into 2 maketh 6, and 6 by 6 maketh 36, whiche multiplied by 7 yeldeth 252. that I set ouer a line for the numeratour, thus: $\frac{252}{}$

Then I multiplye the denominatours 4 by 3 maketh 12, and that by 7 bringeth 84, which multiplied by 9, yeldeth 756, the newe denominatour.

And so the whole reduced fraction is this. $\frac{252}{756}$

 whiche

which is to harde a fraction for me to vnder-stande yet.

Maister. You thinke so, and no meruaile, but anon you shall learne to iudge it easylye, for this fraction is no more in deede then $\frac{1}{2}$, al though it be in greater termes, and therfore more straunger and more obscure.

And this suffiseth for this Reduction, saue that I will shewe you by a figure of measure the iuste rate and reason of this kinde of fractions, and also the dewe vnderstandinge of their Reduction.

The intiere measure parted into 9.

1	2	3	4	5	6	7	8	9	
1	2	3	4	5	6	7	$\frac{7}{9}$		
.1.		.2.			.3.		$\frac{6}{7}$		
1	2	3	4	$\frac{2}{3}$					
1	2	3	$\frac{3}{4}$						

Here you see the longest measure, whiche standeth for the hole and intiere quantitie firste parted into 9 diuisions, whereof 7 are seuered by the seconde measure, and thereof againe are parted out 6. And that 6 being distincte into 3 partes, 2 of them are par-ted by the fourth measure, of whiche fourth measure beynge diuided into 4 partes, the lowest measure dothe containe $\frac{3}{4}$, so that the same $\frac{3}{4}$ muste be truly named, not of the hole measure, but in deede is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$: or as I woulde rather expresse

it, $\frac{3}{4}$ $\frac{2}{3}$ $\frac{6}{7}$ $\frac{7}{9}$

Scholer. This example is so sensible, that I can not choose but see it. And farthermore I see also, that the same fraction is equall to $\frac{1}{6}$ of the intiere measure, as the lines which runne vp and downe doe expressely set forth. Also I see here, that $\frac{2}{3}$ $\frac{6}{7}$ $\frac{7}{9}$ is equal to $\frac{4}{9}$. And farther yet that $\frac{6}{7}$ $\frac{7}{9}$ is equall to $\frac{6}{9}$.

Maist. I am glad that you see it so well, not doubting but you wyll gather greater light of knowledge hereby.

But nowe it is time that we come to the thirde forme of Reduction, whiche treatheth of Improper fractions, that is to say Mixte numbres of vnities and fractions, althoughe they appeare like fractions, as thus $\frac{26}{6}$. whiche dothe include 5 vnityes wholy, and $\frac{1}{5}$ ouer. Wherefore fyrste you shall knowe them be that the numerator is greater than the Denominator.

Scholer. In deede syr that appereth reasonable, that yf the numerator doe expresse more partes to be taken of any vnitie, than the denominator doth signifie that vnitie to be deuided into, it muste nede folow, that such fraction importeth more than ȳ hole, that is to say, the whole with certaine partes ouer. But what Reduction is there in it?

The third forme of Reductiō

Maist. Ther be ij. seuerall kindes of Reduction concerning such Fractions. Some times it shall bee nedefull to conuert those

fractions into the vnities & the proper fraction that will remaine. And somtimes contrary waies, it shall bee meete to reduce mixt numbres, that is vnities written with fractions, into the forme of one symple Fraction, and so be there two wayes.

Scho. What is the meane of the fyrst way to tourne Improper fractions into vnities with their propre fractions?

Reduction of improper fractions into vnities with their propre fractions.

M. That is thus. Your numeratour beyng grater then the denominatour, must be diuided by the same denominatour, and the quotiente therof expresseth the vnities, & remainer shall be put for the numerator of the fraction that resteth, and the denominatour muste be the same that was before

Scho. For example, I take $\frac{17}{5}$. And dyuidinge 17 by 5, the quotient wyll be 3, and there will remaine 2.

Ma. That must you write thus $3\frac{2}{5}$, wher (you se) I haue written 3 without any line, as intier numbres ought to be written, and the 2 that remayned, I haue sett ouer the former denominator with a line, as a propre fraction. And thys number dothe signifye nowe 3 vnities, and $\frac{2}{5}$ of one.

Scho. Then if I woulde by vnities here vnderstande crownes, so it were 3 crownes and $\frac{2}{5}$, that is 2 s.

Ma. Euen so, and therfore $\frac{17}{5}$ dyd sygnifie the same. But this happeneth sometimes that

that when the reduction is ſo wrought, ther remayneth nothing. And then is it not a myxt number, but a ſymple intier number repreſented like a fraction.

Scho. As $\frac{15}{5}$ wyll make 3 iuſte, and $\frac{18}{5}$ wyll make euen 6, This I wyll remembre. But now what is the ſeconde fourme of reduction that you ſpake of for theſe ſortes of fractions?

Maiſter. Whenſoeuer you haue anye of theſe two ſortes of numbres, that is to ſaye, hole numbres without fractions, or whole numbres with fractions, and you woulde tourne them into the fourme of a Fraction, you muſt multiplye the whole number by that denominatour whiche you will haue to remayne ſtyll, and to the totall thereof adde the numeratour whyche you haue all ready. And al that ſhall you ſet for the newe numeratour, kepinge ſtyll the former denominatour: as if you haue $6\frac{3}{4}$, whiche you woulde conuert into an impropre fraction, you muſte multiplye 6 by 4, wherof commeth 24, and therto adde the numeratour whiche is 3, and ſo haue you 27 for the Numerator, and 4 ſtyll for the denominatour.

Reduction of whole numbres of their alone or ioyned with fraction into fractions.

Scho. Then is $\frac{15}{27}$ equall to $6\frac{3}{4}$.

Maiſter. Euen iuſte, and ſo backewarde (as appeareth by the former Reduction) $6\frac{3}{4}$ maketh $\frac{27}{4}$. And thus one of theſe reductions may be the profe of the others

worke.

Scho. This I perceaue, but nowe if you woulde tourne whole noumbres wythoute Fractions into any fraction, I see not howe that may be done, bycause there is no denominatour to make the multiplycation by.

Ma. That was well marked, but this you knowe, that no man intendeth to tourne anye whole numbre into a fraction, but he hathe in his minde that denominatour by whyche the multiplycation muste be made: for the profe wherof I set downe 7, whyche is a whole numbre. And yf you will haue thys numbre conuerted into any certaine Fraction, will me to doe it

Scholer. I praye you reduce 7 into a Fraction.

Mayster. Then you care not what the Fraction bee, so it bee some fraction.

Scholer. Noe I passe not for the sorte of the fraction.

Maister. Then howe canne you thinke that you require me to doe any thynge certaine, whan you leaue mee to dooe as I lyste? And seynge you stande at that stay, whether thinke you that I muste firste intende in mynde what fraction I will make of it, before I cane do it in deede?

Scholer. Els you shoulde dooe ignorauntlye.

Mayster

Master. Then I wyll lymyt my selfe (seeyng you wyll not) to tourne it into quarteres, And therfore I multiplye 7 by 4, (whiche is the denomination of quarters) and there amounteth 28 to bee set for the Numeratoure, and the 4 muste bee sette for the denominatour, and the fraction will bee thus $\frac{28}{4}$.

Schol. In deede I perceaue this to bee resonable, for without muche trial I vnderstād that $\frac{28}{4}$ of any thing, doeth make 7. And soo then if I woulde turne 8 into fift partes, it wyll make $\frac{40}{5}$, whiche is all one with eighte. for 8 crownes tourned into fifte partes, that is into shillinges (will make 40 shillinges, that is $\frac{40}{5}$ of a croune.

Mai. Seing you vnderstande nowe these thre kindes of Reduction, I will declare vnto you the fourth kynde, that is when fractions be written in greter tearmes then theye neede howe they maye be broughte to lesser tearmes.

Scholer. To write any fraction in greater tearmes then nedethe, seemethe to be a faulte, and so this rule seemethe to amende that faulte. The fourth kynde of reductyon.

Maister. It were a faulte to doo anye thynge without nede, whiche after muste bee redressed; but in this case it is not so, nother dyd I saye absolutelye (as you doe) that it nedethe notte to expresse those fractions

in so great tearmes, but that the Fractions doo not neede, I meane for theyr valewe to be vnderstande, but yet it maye be needefull for the ease of those workes whereto theye bee applyed. as for example: In the fyrste kynde of reduction this was youre owne example: $\frac{3}{16}$ and $\frac{4}{6}$, whiche when you woulde reduce, you were faine to tourne them fyrste into odde denomination, and so appeared they thus, $\frac{18}{96}$ and $\frac{64}{96}$, where the Fractions (for theyr owne vnderstandinge) neded not to be tourned out of smaller tearmes into greater, but yet the easynesse of workinge neded it.

Tearmes of Fractions.

Scholar. Syr I vnderstande nowe not only the dyfference of this neede (for the Fractions myghte better bee vnderstanded as Fractions seuerall, eche in his valewe, when they were in lesser Termes, althoughe they coulde not so well be reduced, but also I vnderstande what you meane by greater Tearmes & lesser Tearmes, wherof before I was in doubt, for I see you call the Numeratour and denominatour the tearmes of the Fraction.

Reduction of Fractions into the of terms or Denomination.

Master, I am glade you vnderstande it so well. Nowe than whenne you woulde valewe any Fractions, bycause that maye best bee doone whenne the Tearmes are moste smalest, you shall reduce them to the smalest that you canne, whiche thynge you

may

maye doo thus: Diuide the greateste of anye suche two termes by thee lesser, and if anye thynge remayne, by that remainer dyuide the laste dyuiseur, and if anye thinge remaine now, by that diuide the last diuisoure (whiche was before the remainer of the firste diuision, and so continue styll, till nothinge doo remayne in the diuision, and then marke youre laste diuisour, for it is the number that will easilye reduce youre fraction, if you dyuide bothe the numerator and the denominatoure by the same numbre, and put for the numeratour the quotient of his diuision, and for the denominatour also his quotient that ryseth by his diuision.

S. I take for example $\frac{18}{96}$, & bycause 96 is the greater noumbre, I diuide it by 18, & the quotient is 5, and there resteth 6: what shall I do with this quotient?

M. Nothinge in this worke. But now seyng there remaineth somwhat, by that remayner must you diuide the laste diuiseur.

S. If I shall dyuide 18 (whiche was the laste diuiseur) by 6 that was the remayner, so is the quotient 3, and nothing resteth.

M. As for the quotient, I omitte him yet but bycause there dothe remaine nothinge, therfore is 6 (which was youre laste diuisor) that numbre by whiche you may reduce the fraction proponed.

S. Then as you taught mee, I must diuide the

the numerator 18 by 6, & the quotiente is 3, which I muste put for the numerator ouer a lyne, thus $\frac{3}{}$, and then by the sayd 6, muste I diuide also the denominator 60, & the quotient will be 16, which I must take for the denominator, and so is the fraction $\frac{3}{16}$. And so me thinketh this rule dothe proue the worke of the firste reduction.

Ma. That is true, if the firste reduction wer made of fractions in their leaste termes, and els not without some help, as the second numbre in that place will declare.

S. The second numbre was $\frac{4}{6}$, which was turned into $\frac{64}{96}$, by that rule. Nowe if I shall by thys rule reduce it againe into the leaste tearmes, I must diuide 96 by 64, and there resteth 32 by which 32 I diuide 64, & there remaineth nothing: wherfore I muste take that 32 for the diuisour, to reduce the sayde fraction. Then dose I dyuide 64 by 32, and the quotient is 2. whiche I sette for my numeratour. Agayne I dyuide 96 by 32, and the quotient will bee 3, and so haue I but $\frac{2}{3}$.

M. Muse not at the matter, for you haue done well inough: but you thinke you haue not the fraction that you looked for, that is $\frac{4}{6}$, yet haue you one equal to it, as by ÿ partes of a shillinge you may proofe.

S. Truth it is, for eche of them will bring forth 8 pence, so that $\frac{8}{12}$ and $\frac{4}{6}$ and $\frac{2}{3}$, bee all three

three equall. And nowe I perceiue that by cause $\frac{1}{6}$ was not written in the leaste tearmes that it might bee, therfore thys reduction brought forth not it, but that other whiche is written in the leaste termes. Nowe vnderstande I this rule well. But is there anye other waye to worke this Reduction?

Ma. Yes, but firste note this, that if you finde no suche dyuisor to reduce the fraction tyll you come to 1, bicause 1 dothe make no diuision, therfore that fraction is alreadye in his leaste termes, as by $\frac{81}{100}$ you maye proue, and so of $\frac{81}{98}$, and many other lyke. But nowe to youre question yf you can without that diuision by memorye espye the greateste numbre that may diuided exactlye bothe tearmes of your fraction proponed, then neede you not to vse that diuision. as in this fraction $\frac{60}{96}$, I see that 12 is the greateste numbre that can diuide them bothe: and therfore without any work, by memory only, I turne that into $\frac{5}{8}$, but this abilitie in knowledge is gotten by exercise.

A nother way for to worke Reduction.

Yet, one other way of easye reduction in thys kinde ther is, when your fraction hathe anye cyphers in the fyrste places of bothe tearmes, then may you by castinge away the ciphres, make a bryfe reduction, as thus $\frac{300}{400}$, here take away the ciphres, and it will be $\frac{3}{4}$, which is the same in valewe with $\frac{300}{400}$.

Another waye.

S. And so if I haue $\frac{400}{6500}$, it will bee $\frac{4}{65}$.

Ma.

M. You are deceiued: for you take awaye more cyphares from the numerator, then you do take from the denominator, whiche you maye not do.

S. I confesse my faulte, whiche came of to muche haste. I was more gladder of the rule then wise in vsing it; but nowe I understand it, I trust.

The fyfte kynde of Reductiō

M. Then may I go in hande with the fifte or laste kynde of Reduction, which teacheth how to turne any fractiō proponed into anye other denomination that you lyste: or into any partes, of common coyne, waightes or measures, or such like.

For declaration whereof, firste you shall marke whether youre fraction bee a simple fraction, other els a fraction of sundrye parzes, I meane of more termes then two. And if youre fraction bee a fraction of fractions or otherwaies compound, you muste reduce it to one symple fraction. And then marke well the denomination of that other fraction into whiche you wolde tourne this, for by that denominatour you muste multiply the numerator of youre firste fraction. and the totall thereof shall you diuide by the denominatour of your fyrste fraction. & that quotient shall be the numerator to the denominatour proponed: as for exaumple I haue this fraction $\frac{2}{3}$, whiche I woulde turne into tenth partes, therefore I multiplye this 10 by 2

sh

that is the numeratour of my fraction, and there ryseth 30. whiche I dyuide by 5, and the quotient is 6, whiche muste bee the numeratour to 10. and so $\frac{3}{5}$ will be $\frac{6}{10}$.

Sch. Thys is easy inough to doe.

Ma. Then shall you see an other example of the same fraction that is not so easye: as yf I woulde tourne $\frac{3}{5}$ into viij partes. proue you that worke.

Scholer. I must multiply 8 by 3, and there amounteth 24: whiche I diuyde by 5, and the quotient is 4, then is the newe fractiõ $\frac{4}{8}$.

Maister. And see you nothing doubtful in this worke?

Sc. I see, that when 24 was diuided by 5, there remayned 4, whiche I dyd not passe of, because ye spake nothing of any remayner, but only of the quotient.

M. By lykely hode you remembre what I sayde to you in diuision of whole numbres, that you shoulde not passe of the remayner there, but only note it as a summe that coulde not be diuided with out knowledge of fractions. Wherfore nowe marke thys, that in all diuisions of whollе numbres, when there is any remayner, you shall set it ouer a lyne as a numeratour, and set the diuysor for the denominatour, and that fraction doth make the diuision complet, and is part of the quotient: as if I woulde diuide 48 by 5, the quotient will be 9 $\frac{3}{5}$: so in your former worke

 when

when 24 was diuided by 5, the quo-
tient shoulde bee $4\frac{4}{5}$ and so the newe $\frac{4\frac{4}{5}}{8}$
fraction shoulde be thus.
that is $\frac{4}{5}$ of the intiere numbre, and
$\frac{4}{5}$ $\frac{1}{8}$, whyche you may proue by exaumple of
some coyne.

Scholar. Then I take a Crowne, whose
$\frac{1}{5}$ is 3 d. Nowe if I woulde proue
whether 3 d bee, I shall haue a $\frac{\frac{4}{5}}{8}$
combrouse worke to doe.

M. In deede for whole pennies your exã-
ple is troublesome: yet tourning the crowne
into halfe penies, it is easy ynough.

Scholer. Nowe will I do it.

M. Firste let me tell you an easy way how
to fynde any numbre, that wyll easily be dy-
uided into suche partes as you desire, whiche
way is thys. Set downe the parts that you
desire, and then by one of them multiply al
the other, the totall whereof shall containe
all the parts proponed: As if I woulde
haue a nūbre that may be diuided into 4 5 6
and 7 partes, by 4 multiply 5, and there ry-
seth 20: then multiply 20 by 6, and it will
make 120: which multiplied by 7, will yelde
840: and so of any other numbres.

Sc. Then in our former example where is
mention but of 5 partes & 8 partes, I shal
only multipy 5 by 8, whiche maketh 40, &
that numbre will serue.

Maister. So will it.

Sch

Scho. Then what is $\frac{3}{5}$ of 40?

M. Proue by the same rule which you co-
fesse easy ynough; 3 times 40, is 120, which
beyng diuided by 5, maketh 24 $\frac{4}{\frac{4}{5}}$ / 8
and that in iuste.

Nowe to knowe whether be equall to 24,
first I se by the same rule, that $\frac{4}{8}$ is 29, and
$\frac{1}{8}$ is 5, of which 5 I must take $\frac{4}{5}$:
and that by the same rule is 4. $\frac{4}{5}$
So that I see nowe, that is e- $\frac{4}{8}$
quall to $\frac{3}{5}$.

Mast. And by the way note this form of fraction how it is written, that is to say, both the Numeratour and his fraction aboue the lyne. Although I know it may bee writ-ten eyther waies, as thus $\frac{4}{8}$ and $\frac{4}{5}$ $\frac{1}{8}$, but I accompte the other waye more apte a greate deale.

And so may you expresse by an other way (then is before mentioned) al frac-
tions of Fractions, as thus. $\frac{3}{4}$
that is $\frac{3}{4}$ of $\frac{5}{8}$, and so of other, $\frac{5}{8}$
but I remitte these fourmes to
the arbitrement of euery wise artes man, to vse as he thinketh most apte and ready.

But now one example more for this rule, and then shall we ende it. If I haue $\frac{7}{15}$ of a Soueraigne (accomptinge ẏ Soueraygne 20 shillinges) howe many shillinges is that $\frac{7}{15}$?

Scho. I muste multiply 7 by 20, & that maketh 140, whyche I shall diuide by 15,

 and

and the quotient wyll be $9\frac{5}{135}$, or els in lesser termes $\frac{1}{5}$.

M. That is 9 ß and 4 d. And this for this time shall suffice for Reduction, saue that I must now repeate a little touchinge Multiplication and Diuision, and so go forwarde.

MVLTIPLICATION.

IN Multiplication it happeneth somtime ỷ ther be hole nūbres to be multiplied with fractiōs, and may bee in two sortes, for other the whole nūbre is seuerall from ỷ fraction, and is the multiplier, or els ỷ whole numbre is ioyned with one or both of ỷ fractions, and so maketh a mixt numbre therof. If it bee in the firste sort, then nedeth there no reduction, but only multiply the numeratoure of the fraction by that whole number, and the totall therof set for the newe numerator.

Reductiō of whole numbres into fractions.

S. I vnderstand you thus. If I haue $\frac{6}{23}$ to be multiplied by 16, then muste I multiplye that 16 with 6, whiche is the numetor, wherof commeth 96, and that muste I set for the newe numerator, keepynge styl 23 for the denominator, and so the fraction will be $\frac{96}{23}$, that is $4\frac{4}{23}$.

Mast. And in this sort of worke you maye abbredge the labour thus: If it happen the deno-

denominator to bee suche a numbre, as may euenlye bee diuided by the sayde whole numbre proposed, then dyuide it therby, and sette the quotiente of that diuision for the former denominatour: but reserue styll the numerator, and so is the multiplication done.

Scholer. Then I fayne this example, $\frac{7}{20}$ to bee multiplied by 5. And bycause 5 will iustlye diuide 20, therfore I take the quotient of that diuision whiche is 4, and sette in steed of 20, and so the fraction will bee $\frac{7}{4}$, that is $1\frac{3}{4}$.

M. Whiche is alone with $\frac{35}{20}$ that woulde haue folowed of the other sorte of worke.

Sc. I perceaue it very wel.

Maister. Nowe then for the other sorte where the numbre is mixt, take this waye: fyrste to reduce the sayed whole numbre and fraction into one fraction impropre, (as I shewed you in Reduction) and then multiply theym together, as if they were propre fractions. Howe to multiplye mixt numbres.

S. $13\frac{3}{5}$ beinge set to be multiplyed by $\frac{5}{8}$, fyrste I muste reduce the myxte numbre by multiplying 13 by 5, and that maketh 65, wherto I muste adde the numerator 3, and so the fraction will be $\frac{68}{5}$, which now I shall multiply after the accustomed forme, and it wyll be $\frac{340}{40}$.

M. You haue done well, and so may you see, that althogh moste parte of the formes

of multiplication may bee wroughte without Reduction, yet some can not, as namely Mixed numbers.

And yet one note more will I tell you of Multiplication, before we leaue it: That is, when so euer you woulde multiply any fraction by 2, which commonly is called Duplation, you maye do it not only by doubling the numeratoure, but also by partinge the denominatour into halfe, if he be euen.

Duplation.

Sc. Then if I woulde double $\frac{5}{12}$, I maye choose whether I will make it, $\frac{10}{12}$, or els $\frac{5}{6}$. And in deede I see that all is one, but that the diuydinge of the denominatour seemeth the better waye to make smaller tearmes of the fraction, and so they shall neede the lesse reduction.

Ma. It is so: and nowe I shall not neede to tell you that Multiplication is proued by Diuision, and Diuision lykewaies by Multiplication, but the lyke workes that I shewed you in Multiplication, wyll I shew you in Diuisyon also.

Diuision to diuide a whol nũbre by a Fractiõ

When any whole numbre shall bee diuided by a fraction, you muste multiplye the sayde whole numbre with the denominator of the fraction, and set the totall thereof for the newe numerator: and for the denominatoure set the numeratour of the fraction.

Sch. Then 20 diuided by $\frac{3}{4}$ will make $\frac{80}{3}$.

Mast. Euen so. But if you woulde dyuide the

the fraction by the whole numbre, then multiplye the denominatour by the same whole noumbre, and set that totall for the denominatour, without changinge the numeratour. To diuide a fraction by a whol numbre.

Scholer. Then to dyuide $\frac{20}{23}$ by 4. it wil be $\frac{20}{92}$.

Ma. You say well. And by the same example you geue me occasion to remembre an other brefer waye to doe the same. For if you had diuided the said numerator by 4, and set the quotient for the numerator, kepinge styll the olde denominator, it wolde haue been not only as well done, but also in a fraction of lesser termes. An other breif way

S. I gesse it to be euen so, by a lyke work that you taught me in multiplication. And for proofe therof $\frac{20}{23}$ beynge the diuidend, and 4 the diuisor, I diuide the numerator 20 by 4, and the quotient is 5. which I set for 20 ouer 23, thus $\frac{5}{23}$. And I see that it is all one with $\frac{20}{92}$, as by dyuiding both these tearmes by 4, and so reducinge them to their leaste denomination, I maye easily proue.

M. You conceaue it wel. And if there be mixt numbres (other one or both) you muste fyrste reduce that mixte numbre into an improper fraction. And then work as you haue learned.

S. That was sufficiently taught in Multiplication. Therefore I pray you go forward to some other thinge.

M. Then take this note yet for Diuision.

If the denominators be like, then diuide the numerators as if it were in whole numbres, & the quotient, whether it bee fraction, whole numbre or mixte, is a good quotiente for that diuision. And generally, yf one of the numerators may iustlye diuide the other, by that quotiente multiplye the denominatour of the lesser numerator, and set it that dothe amount in the roome of the same denominator, and then for a numerator to it, set the denominator of the other fraction.

S. Then if I woulde diuide $\frac{3}{4}$ by $\frac{12}{17}$ I see that 3 will diuide 12, & the quotient will be 4, by whiche I must multiplye the other 4, that is the denominatour vnder 3, and then is it 16, whiche I sette for the denominator 4, and ouer it in steede of the 3, I muste sette 17, the other denominator: and so is it thus, $\frac{17}{16}$.

Ma. And so is $\frac{17}{16}$ in steede of $\frac{51}{48}$, whiche woulde haue rysen by the common worke.

Mediation. And now for Mediation (whiche is to diuide by 2) marke this: If the numerator bee euen, set the halfe of it in his place without the diuisour: and so haue you doone. and if the numeratour bee not euen, then double the denominatour.

S. That is, if I wolde mediate $\frac{6}{11}$ I maye make the quotient $\frac{3}{11}$. And if I woulde mediat $\frac{7}{11}$, I must make it $\frac{7}{22}$.

Maister. Now truste I that you haue sufficient

ent knowledge in Reduction, Multiplication, and Diuision: And therfore will I goe in hande with Addition and Subtraction, which now will appear easy inoughe.

ADDITION.

Whensoeuer you haue any fractions to bee added, you muste cōsidre whether thei be of one denomination or not. And yf they bee of one denomination, then adde the numerators together, and set that amoūteth for the numerator ouer the common denominator, & so haue you done. The reason is, bicause ÿ suche differ litle in Addition or Subtraction from the worke of vulgar denominations, where the denominatours bee noe numbres: as 3 pence and 5 pence, make 8 pence, where the denomination is not altered. But and if the fractions bee not of one denomination, or any of theym bee mixte of whole numbres and fractions, then muste you firste reduce them to one denomination, and after adde them. And if they bee manye, then adde firste twoo of theim, and to the summe that dothe amount of that Additiō, adde the thyrde, & then the fourth, and so forth, yf you haue so many.

Scholer. This seemeth easie ynoughe nowe that I haue alreadye learned to mul-

tiplye and to Reduce, without whiche two, I coulde neuer haue wrought this. And therfore nowe I see good reason, why you dyd place Multiplication and Reduction before Addition.

Ma. It is well considered, but yet refuse not to expresse your understandinge of yt by an example.

Scholar. Then woulde I adde fyrste $\frac{7}{18}$, with $\frac{5}{18}$, and bycause the denominatours are lyke (and so needeth no Reduction) I adde 7 to 5, whiche maketh 12, and then is my summe $\frac{12}{18}$, that is in smaller numbres $\frac{2}{3}$.

And if I haue many numbres to bee added, as here $\frac{3}{8}$ $\frac{4}{5}$ $\frac{9}{10}$, fyrste I muste reduce them (bicause they haue dyuers denominatours) into one denomination, and then wyll they bee thus.

$$\frac{150}{400} \qquad \frac{320}{400} \qquad \frac{360}{400}$$

or in lesser tearmes $\frac{15}{40}$ $\frac{32}{40}$ $\frac{36}{40}$, whiche by addition do make $\frac{83}{40}$, that is $2\,\frac{3}{40}$.

Maister. Nowe maye wee goe to Subtraction.

SUbtraction hath the same precepts that Addition had: for if the denominators be like, then muste you subtracte the one numeratour from the other, and the rest is to bee sette ouer the common denominatour, and so is youre Subtraction ended: but and if you haue manye Fractions to bee subtracted out of manye, then muste you reduce them to one Denomination, and into twoo seuerall fractions, that is, all that muste bee subtracted into one fraction, and the residue into an other fraction, and then worke as I sayde before.

Scholar. For the firste exaumple I take $\frac{5}{12}$ to bee subtracted out of $\frac{7}{12}$, and the reste wil bee $\frac{2}{12}$, or $\frac{1}{6}$.

For an other exaumple I take $\frac{3}{4}$ to bee subtrahed out of $\frac{7}{8}$, whiche I muste fyrste reduce, and it will bee thus, $\frac{24}{32}$, and $\frac{28}{32}$.

Then doe I subtracte 24, oute of 28, and there resteth 4, which I sette ouer the common denominator for a remainer, thus. $\frac{4}{32}$, that is $\frac{1}{8}$.

Now for the thirde exaumple, I take $\frac{3}{4}$ and $\frac{5}{6}$ to be subtrahed from $\frac{7}{8}$ and $\frac{9}{10}$. And bycause theire denominatours bee diuers, I doe reduce them thus, $\frac{1440}{1920}$ $\frac{1600}{1920}$ $\frac{1680}{1920}$ $\frac{1728}{1920}$.

Then doe I adde the twoo fyrste, and they make $\frac{3040}{1920}$. Also I adde the twoo laste

and

and they yelde $\frac{3408}{1920}$. Then doe I subtracte 3040 out of 3408, & there resteth 368, so is the remayner $\frac{368}{1920}$, that is in smaller termes $\frac{23}{120}$. And thus haue I done with Subtraction, excepte you haue anye more to teache mee.

M. Proue one example more of two fractions of dyuers denominations.

S. I take these two fractions, $\frac{7}{8}$ and $\frac{9}{24}$, whiche beinge ruduced, will stande thus, $\frac{168}{192}$, and $\frac{72}{192}$. Now woulde I subtract 168 out of 72, but I can not.

M. Then may you perceaue that you mistoke the fractions, for you can neuer subtract the greater out of the lesser, althoughe you may adde, multiply or diuide the greater with the lesser. And albeit that $\frac{7}{8}$ hathe bothe hys termes lesser then $\frac{9}{24}$, yet is $\frac{9}{24}$ ye lesser fraction: for generallye if you multiplye the numeratours and denominatours of two fractions crosse waies, that fraction is the greatest, of whose numeratour commeth the greatest sum: as in this example: 7 multiplyed by 24, maketh 168: & 9 being multiplyed by 8, yeldeth but 72, therfore is the first fraction $\frac{7}{8}$, the greatest of these twoe, so canne you not subtract it out of a lesser fraction.

The greateste of two fractions.

But and you shoulde subtracte a fraction out of a whole numbre, what wolde you doo?

S. Marry I wold reduce that whole numbre into a fraction of the same denomination,

that

that my fraction is, and then work by Subtraction.

Master. So may you doo, but it is easyer muche if your fraction bee a proper fraction, that is to saye, lesse then an vnitee, to take an vnitie from the whole numbre, and then tourne it into an Improper fraction, and so worke youre Subtraction. As if I wolde subtracte $\frac{1}{5}$ from 4, I maye take one from 4 and tourne it into $\frac{5}{5}$, from whiche yf I abate $\frac{1}{5}$, there wyll remayne $3\frac{4}{5}$. And if the fyrste fraction be an improper fraction, then maie I take soo manye vnyties from the whole numbre, that they maye make an improper fraction greater then that fyrste, and then worke the subtraction. As if there bee proponed $\frac{10}{3}$ to be subtrahed from 6, because $\frac{10}{3}$ is more then 3, and not 4, I muste take 4 from 6, and tourne them into thyrdes, thus. $\frac{12}{3}$, then abate $\frac{10}{3}$, and there resteth $\frac{2}{3}$, so the whole remainer is $2\frac{2}{3}$. And thus wyll I make an ende of the woorkes of common fractions for thys time, not doubtinge, but you can applie them bothe vnto the rules of Progression, and also vnto the Golden rule without any other teachinge then you haue learned before, whyche myghte seme tedyous to repete, saue that in some speciall diuersities, whiche be peculier to fractions, I can not ouerpasse, but instructe you somewhat by the waye.

The

THerefore as touchinge yͤ Gol-de rule for the placinge of th iij, numbres propoũded in th questiõ, wherby to find yͤ thir & for yͤ forme of their worke w other like notes, I referr you to yͤ whiche you haue already learned.

But this easy forme of working by fra-tions shall you note, yͭ if your thre nũbres b fractions for an apte worke & certaine, mul-tiply yͤ numerator of the firste numbre in th questiõ, by the denominatour of the seconde & all that againe multiply by the denomina-tour of the thirde nũbre: and yͤ totall thereo shall you kepe for to be the diuisour. The multiply the denominatour of yͤ first nũbre b the numeratour of the seconde, & the whol therof by the numerator of the thirde, & th totall therof shall be your diuidend. Nowe di-uide this diuideđ by the diuisour whiche yo founde out before, and that numbre shal be th iiij numbre of the question, whiche you seek for: As in this example: If $\frac{3}{4}$ of a yarde o veluet coste $\frac{2}{3}$ of a Soueraigne, (estemed a 20 shillinges) what shall $\frac{1}{6}$ cost?

A questi-on of Vel-uet

Scho. If it please you to let me make th answere, I wolde firste place these thre num-bres, as I learned in whole numbres, thus.

$\frac{3}{4}$ $\frac{2}{3}$

$\frac{1}{6}$

An

And then according to youre newe rule I muste multiply 3, being numeratour in the fyrste numbre, by thre the denominatour of the seconde, and therof commeth 9, which I multiply agayne by 6, the denominatour of the third numbre, and so haue I 54, which I kepe for the diuiser, then multiplye I 4, the denominator of the syrste, by 2 the numeratour of the seconde, and there ryseth 8: whiche againe I multiply by 5, the numeratour of the thirde, and it maketh 40. then muste I dyuide 40 by 54, and it wyll be $\frac{40}{54}$ that is $\frac{20}{27}$, in lesser tearmes, and then the figure wyl stand thus.

$$\begin{array}{ccc} \frac{3}{4} & & \frac{2}{3} \\ \frac{5}{6} & & \frac{20}{27} \end{array}$$

But what that is in mony, I can not tell, except I shall worke it by Reduction, as you taught me.

M. It forceth not nowe, you may reduce it when you lyst, but it were disorderlye doone here to mingle diuers workes togythere, where we do not seke the valew of the thing in common monye, but in an apte numbre, which you haue well done. And therfore will I yet shew you an other like way of easines in worke, howe you maye chaūg your iij fractions into 3 whole numbres, by whiche you shall worke as yf the question were proponed in whole numbres. The fyrste numbre you shall fynde, as I taughte youe.

nowe

now to finde the diuisor, for the seconde num-
ber take the numeratour of the seconde frac-
tion: and for the thirde numbre take that, tha[t]
ryseth of the multiplication of the denomi-
natour of the firste, by the numeratour of th[e]
thyrde, and then worke your question.

A questi-
on of
Siluer.

Sc. For exãple here of, I put this questiõ,
If $\frac{11}{12}$ of 1 li waight of siluer, be worth $\frac{12}{4}$ o[f]
a souerain, what is $\frac{1}{2}$ of 1 li. weight worth?
For the aunswere, firste I
place ŷ fractiõs in order, thus.

$\frac{11}{12} \; Z \; \frac{12}{4}$
$\frac{1}{2}$

Then to tourne these fractiõs
into whole numbres, I multiply 11 which
is the numeratour of the firste, by 4, (the de-
nominatour of the seconde) and there cometh
44, whiche I multiply by 2, the denomina-
tour of the thirde, and so amoũteth 88, whi-
che I set for the diuisour in the firste place.
Then in the seconde place I set 12, which is
numeratour in the seconde fraction, and in
the thirde place I sette the summe that a-
mounteth of 12, beinge the denominatour
in ŷ firste numbre, multiplied by 1, being nu-
merator in ŷ third nũbre, & so ŷ
figure wyll stande as here you se.

88 Z 1[2]
12

Thẽ to worke it forth, I multi-
plye 12 by 12, and there amounteth 144
which I diuide by 88, and the quotient wil
be $1\frac{56}{88}$, or in lesser termes. $1\frac{7}{11}$ & then th[e]
figure will stand thus.

$\frac{11}{12} \; Z \; \frac{12}{4}$
$\frac{1}{2} \quad 1\frac{7}{11}$

M. These ij. fournes now you

vnderstande well ynoughe: And as for anye o-ther, at this time I will not repeate, onelye this shall you marke for the proofe of thys rule, whether your worke be well wrought or no. Multiply the firste noumber by the fourth, and note what amounteth: then multiplye the seconde by the thirde, and marke what amounteth also. Nowe yf those twoo numbers so amounting, bee equall, then is you worke well done, elles you haue erred. And this shal suffice for the former rule. But in the Backer rule, this shall you note for ease of worke, that you multiply the numeratour of the firste, by the Numeratour of the seconde, and the whole there of, by the denominatour of the thirde, and that amounteth therof, shall be the diuidende. Then multiplie the denominatour of the fyrste, by the denominatour of the seconde, & that whole by the numerator of the thyrde, & that riseth therof, shall be the diuisor Example of thys: I did lende my frinde $\frac{3}{4}$ of a Porteguise vij. monethes, vpon promise that he shoulde doe as muche for me againe, and when I shoulde borowe of him, he could lende me but $1\frac{5}{2}$ of a Portegruise, nowe I demaunde, howe longe time muste I keepe this money in iuste recompence of my lone, accompting 13 monethes in the yeare?

The proofe of the golden rule

The Backer rule.

A question of lone

Scho. The fyrste number muste be the firste money borowed, that is $\frac{3}{4}$ of the Protegui-

guise: the second number ẏ 7 monethes that is $\frac{7}{12}$ of a yeare, and ẏ thirde number the mony that was lent in recompence, that is $\frac{5}{12}$ of a Porteguise, then I sette the noumbers thus.

$$\frac{3}{4} \quad \frac{7}{12} \quad \frac{5}{12}$$

Then (as you taught me) I multiply 3 (beyng numeratour in the firste number) by 7 the numeratour of ẏ seconde number, and it maketh 21, whiche I multiply by 12 the denominatour of the thirde, and so haue I 252 for the diuidende: then I multiply 4, the denominatour of the firste, by 13 the denominatour of the seconde, and it yeldeth 52, whiche I multiplye againe by 5, the numeratour of ẏ thirde, and it will make 260 that is the dyuisour. Then muste I diuide 252 by 260, so it wyl be in the smalleste Fraction $\frac{63}{65}$ of a yeare. M. And thus doe you see somme ease in woorking better, then to multiplye and diuide tediouslye so manye Fractions. Another question yet will I propose, to the intent you may see therby the reason of the Statute of assise of breade and ale, whiche in all statute bokes in Frenche, Latine, & Englishe is muche corrupted for want of knowledge in this arte: for the right vnderstandinge wherof I propone this question.

Statute of Assise of breade and ale

A question

When the price of a quarter of wheate is 2 s. the farthinge white lofe shall way 68 £, the

I demaunde: What shall suche a lofe waye, when the quarter of weate is solde for 3 s?

Scholer. This question muste be wrought as it is proponed, in whole numbers, and not in Fractions.

Ma. You seme to say reasonable, how be it in that Statut of Assise, the rate is made by ÿ proportion of partes in a pounde weight Troye els coulde it not be a statute of anye long continuaunce, seinge the shillinges doe chaunge often, as all other monies doe: but this statute well understande, is a continuall rule for euer, as I will anone declare by a newe table of Assise, conuertinge the shillinges into ounces, and partes of unces.

Therfore here by a shillinge you muste understande $\frac{1}{20}$ of a pounde waight, and so by pens $\frac{1}{20}$ of a unce, wherefore althoughe you might woorke this question proponed by whole noumber weell ynough for that tyme when the statute was made, yet to applye it to our time, and to make it serue for all times generallye it is beste to woorke it by fractions, setting for ij. shillings $\frac{2}{20}$: and for 68 shillinges, $\frac{68}{20}$: and so for three shyllinges $\frac{3}{20}$ and then wyll the figure of the question stand thus. — Note what a shilling is

$$\frac{2}{20} \qquad \frac{68}{20}$$
$$\frac{3}{20}$$

In which question, bicause all the denominatours be lyke you shall woorke onelye wyth the numeratours.

Scholer. Then I shall multiply 68 by 2 wherof commeth 136, whiche yf I diuide by 3, the quotient wilbe 45 $\frac{1}{3}$: but howe shal I make a Fraction of that to stande with the other?

Maist. Haue you so sone forgoten what was taught you so lately? Thys ys hys fourme. $\frac{45\frac{1}{3}}{20}$

Scho. I remember it nowe, & then it signifieth 45 twenty partes, and the third deale of one twenty part.

Maist. So is it. and that maketh in shyllinges, 45 shillinges, four pens, wherby you may note one great erroure in the Statute bookes, which haue constantly 48 shillinges in that assise. And by this rule, if you examine the statute, you shall finde many summes false, wherfore for the true vnderstanding of that statute and suche like as I haue made mention of yt, and somewhat recognised yt, so doe I wishe that all gentlemen and other studentes of the lawes, wolde not neglecte this art of Arithmetike as vnnedefull to their studies. Wherefore to encourage them therto, and to greatifye both them and al other in generall, I will here exhibit a table of that part of ŷ statute in two columes, and in a third columme I will add the correction of those errours whiche haue crept into yt

Here foloweth the Table.

When a quarter of wheat is solde for

The price of a quarter of wheate.		The weight of a farthing white lofe by the statute bookes.			The correction by iust assise.		
s.	d.	li.	s.	d.	li.	s.	d.
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	3	8	0	3	8	0
2	6	2	14	4 3/5	2	24	4 4/5
3	0	8	8	0	2	5	4
3	6	2	2	0	1	10	10 2/7
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 2/3
5	0	1	8	2 1/3	1	7	2 2/5
5	6	1	4	8 1/4	1	4	8 8/11
6	0	1	2	8	1	2	8
6	6	0	19	11	1	0	11 1/13
7	0	0	19	1	0	19	5 1/7
7	6	0	18	1 1/2	0	18	1 3/5
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 1/4	0	15	1 1/3
9	6	0	14	4 3/4	0	14	3 15/19
10	0	0	13	7 1/2	0	13	7 1/5
10	6	0	12	11 1/4	0	12	11 3/7
11	0	0	12	4 1/4	0	12	4 4/11
11	6	0	11	10	0	11	9 71/23
12	0	0	11	4	0	11	4

In the cōmon bokes there is no farther rate of assise made, then vnto 12 s the quarter of wheate, but in an anciēt copie of 200 yeares oulde (which I haue) ther is added the rate of assisse vnto 20 s the quarter, but yet was ẏ assisse also eyther wrong cast at ẏ first pening, or els corrupt sith that time, for lacke of iust knowledge in the rule of proportion, whych I wil adde here also, to gratifye such as be studious in thys sorte of knowledge, and desire to vnderstande truth exactly.

The price of a quarter of wheate.		The weight of the farthing white lofe by ẏ statute bokes.		The correction of the errours		
s.	d.	s.	d.	s.	d.	
12	6	11	0	10	10	14/25
13	0	15	0 ½	10	5	7/13
13	6	10	1 ½	10	0	8/9
14	0	9	7	9	8	4/7
14	6	9	2 ½	9	4	16/29
15	0	9	1 ½	9	0	4/5
15	6	9	1 ¼	8	9	[illegible]
16	0	9	0	8	6	[illegible]
16	6	8	6	8	2	[illegible]
17	0	8	3	8	0	[illegible]
17	6	7	10	7	9	[illegible]
18	0	7	6	7	6	2/3
18	6	7	3	7	4	[illegible]
19	0	7	2	7	1	[illegible]
19	6	5	10	6	11	[illegible]
20	0	5	6	6	9	3/5

These ij tables I haue set seueral, bicause no man should thinke that I woulde eyther adde or take away from any lawe those partes whiche might of right seme eyther superfluous other diminute, but yet I may not bee so curious as to neglect manifeste errours, which is not onely my parte, but euery good subiectes duetie with sobrietie to correct. And for auoyding of offence, I haue rather done it in this priuate booke, rather then in any booke of the Statutes selfe, trustinge that all men will take it in good parte.

Scho. I woulde wishe so, but I dare not hope so, sith neuer good man that woulde reforme errour, coulde escape the venimous tungues of enuious detractors, whyche bycause they either can not or liste not doe any good them selues, doe delite to barke at the doyngs of other. But I beseke you to staye nothinge for their peruerse behauiour.

Ma. I consider many thinges that some may obiecte, whereunto I am not vnprouided of iust aunsweres, but I will not seme so hasty to make the answers before I heare their obiectiõs: but as I truste that men are of a better nature, & more gratefull nowe then some hath been in times passed, as I haue done in the statute of assyse for breade in rate of shillinges, so will I set forthe the like table in poundes & ounces, and the parts therof, that it may bee easily applied to all times:

 but

but I meane not by this toe alter any worde of the Statute (being so good an ordinaunce and of so greate continuance) but onelye to make it as a kinde of exposition & declaracyon of the said Statute, trusting that thereby the Statute may be better vnderstande, and consequently better put in execution. And here you shall note, that I haue accompted ỹ shillinges after the rate of xx. ß. to the pounde weight, bycause I esteme it the moste apte rate for our time. Wherefore if in the fyrste columne you find ỹ price of wheate, directly againste it in the seconde columne you may finde the weight of the farthinge whitelofe, in this our time: and if you double that number (as I haue done in the thirde columne) then haue you the weight of the halfe penny white lofe: and so in the fourth columne is set the weight of the penye white lofe. It nedeth not to tell you that, that the syghte dothe testifie howe that euerye columne is parted into thre smaler pillers, where of the firste colum hath these thre titles, poundes, shillinges, and pennies: the other three columes, haue eche of them these three tytles, Poundes, Vnces, & Penny weightes. And as in the first colum xij. pence make shillinge, and xx. ß. maketh a pounde, so in the other iij. columes, xx. penny weight maketh an vnce, and twelue vnces doe make pounde.

A pound weight.

Th

[P]rice of a quarter of wheate.			The weight of a farthing white lofe.		
l	s	d	li.	unc.	d. w.
0	3	0	6	9 1/2	2
0	4	6	4	6 1/4	3
0	6	0	3	4 3/4	1
0	7	6	2	8 1/2	2 4/5
0	9	0	2	3	4
0	10	6	1	11 1/4	1 2/7
0	12	0	1	8 1/4	3
0	13	6	1	6	2 2/3
0	15	0	1	4 1/4	1 2/5
0	16	6	1	2 3/4	1 8/11
0	18	0	1	1 1/4	2
0	19	6	1	0 1/2	1 1/13
1	1	0	0	11 1/2	3 1/7
1	2	6	0	10 3/4	2 3/5
1	4	0	0	10	4
1	5	6	0	9 1/2	2
1	7	0	0	9	1 1/3
1	8	6	0	8 1/2	1 15/19
1	10	0	0	8	3 1/5
1	11	6	0	7 3/4	0 3/7
1	13	0	0	7 1/4	3 4/11
1	14	6	0	7	1 21/23
1	16	0	0	6 3/4	1
1	17	6	0	6 1/2	0 14/25
1	19	0	0	6 1/4	0 7/13
2	0	6	0	6	0 8/9

The weight of ȳ halfpennie white lofe.

li.	unc.	d.w.
13	7	4
9	0 3/4	1
6	9 1/3	2
5	5 1/4	0 3/5
4	6 1/4	3
3	10 1/2	2 4/5
3	4 3/4	1
3	0 1/4	0 1/3
2	8 1/2	2 4/5
2	5 1/2	3 5/11
2	3	4
2	1	2 2/13
1	11 1/4	1 3/7
1	9 3/4	0 1/5
1	8 1/4	3
1	7	4
1	6	2 2/3
1	5	3 11/19
1	4 1/4	1 2/5
1	3 1/2	0 6/7
1	2 3/4	1 8/11
1	2	3 19/23
1	1 1/2	2
1	1	1 3/25
1	0 1/2	1 6/13
1	0	1 7/26

The weight of ȳ penie white lofe.

li.	unc.	d.w.
27	1/4	3
28	1 1/2	2
13	7	4
10	10 1/2	1 1/5
9	0 3/4	2
7	0 1/3	0 2/3
6	9 1/2	2
6	0 1/2	0 2/3
5	5 1/4	0 3/5
4	11 1/4	1 10/11
4	6 1/4	3
4	2	4 4/13
3	10 1/2	2 4/7
3	7 1/2	0 2/5
3	4 3/4	1
3	2 1/4	3
3	0 1/4	0 1/3
2	10 1/4	2 3/19
2	8 1/2	2 4/5
2	7	1 5/7
2	5 1/2	3 1/11
2	4 1/4	2 15/23
2	3	4
2	2	2 6/25
2	1	2 2/13
2	0	3 5/6

Schol. Sir I doo thanke you most hartilye for thys, not onely in myne owne name and in the name of all studientes, but also in the name of the hole commons, to whome the restitution of this assise (I trust) shall bringe restitution of the weighte in breade, whyche longe tyme hathe been abused. And yf you knowe any lyke thynges more, wherein you wolde vouchsafe to declare the erroures, and set forth the truthe, you can not but obtayne greate thankes of all good harted men that loue the common wealth.

M. I haue sundry thinges to declare, but I haue reserued them for a priuate booke by yt selfe, yet notwithstandinge bicause the statute of the rate of measuringe of grounde is so common that it toucheth all men, and yet no more common then needeful, but so muche corrupt, that it is to far out of all good rate, not only in the Englishe bookes of statutes commonly printed, but also in the latine bookes, and in the Frenche also: for I haue reade of eche sorte, and conferred them diligently, I wyll giue you a Table for the restitution of those errours, as may suffice for this presente tyme. And fyrste wyll I propose one question to you touchinge the vse of that statute, whereby you may perceaue the order howe to examine the whole statute and euery parcell thereof, and the question ys thys. When the aker of grounde doothe contayne A question of measure of Ground.

foure

foure perches in bredth , than muste it conteyne 40 perches in lengthe : thenne doe I demaunde of you, how much shall the lengthe of an Acre bee, when there is in the breadth of it 13 perches? But before you shall aunswere to this question , I will declare vnto you an other statute, whiche is the grounde of the former statute . And that statute ys this: It is ordeined, that 3 Barly cornes, drie and rounde, shall make vp the measure of an ynche: xii. ynches shall make a foote: and iii. foote shall make a yearde: (the common Englyshe bookes haue an elne) fyue yeardes and a halfe shall make a perche : and fortie perches in lengthe , and iiij. in breathe , shall make an acre. This is that statute , whereby you maye perceaue , that the intent of the statut is, that one acre shoulde containe 160 perches. Nowe let me heare your aunswere to the question.

A statute of measures

Scholer. As I perceaue by the wordes of that Statute, a perche to bee $\frac{1}{160}$ of an aker, so coulde I make those numbres all in fractions, and so worke the question: but seeynge I maye doe it also in whole numbres, I take ẏ forme for ẏ most easy, therfore thus I set the questiō in forme: thē do I multiply 40 by 4 & it maketh 160, which I diuide by 13, and the quotient is $12\frac{4}{13}$

$$\begin{array}{ccc} 4 & & 40 \\ 13 & & \end{array}$$

Ma. Nowe tourne that $\frac{4}{13}$ into the com-

mon

mon partes of a perche, as they bee named in the former statute; howbeit, it shall be best to take one of the leaste partes in denomnation for auoydinge of muche laboure, as feete, whereof the perche containeth $16\frac{1}{2}$.

S. Than to tourme $\frac{4}{13}$ into feete, I multyplye $16\frac{1}{2}$ by 4, and it maketh 66, which I muste diuide by 13, and the quotient is $5\frac{1}{13}$

Ma. So I fynde, that if the acre hold in bredth xiii. perches, it shall containe in length 12 perches, 5 foote, and $\frac{1}{13}$ of a foote, which is not fully an ynche, for the ynche is $\frac{1}{12}$ of a foote. But heere all the statute bookes in latine and englishe (that I haue seene) dooe note to bee 13 perches, 5 footh, and 1 inche, which maketh aboue xiii. perches to manye in the acre, so that I woulde haue thought the erroure to haue crepte into the prynted bookes by the gerat neglygence that printers in oure tyme do vse, saue that in wrytten coppyes of greate antiquitie, I doo fynde the same. Yet haue I one frenche coppie, whiche hath 12 perches $\frac{1}{4}$, and one foote, and that mysseth very lyttle of the truthe.

Scho. Then I se it is true that I haue often heard say, that the truest copies of the Statutes, be the frenche coppies.

M. That is often true, but not generally, as I haue by conference tried diuersly: but in this statute the frenche boke is moste corrupt in all other places lightely.

But

But now to perfourme my promyse, I wil sette forth the table for measuringe of an aker of grounde only by such partes as the statute doth mention, bycause at thys tyme I dooe of purpose write it for the better vnderstandinge of the statute, and herafter wyth other thinges, I entende to set foorth thys same more at large.

In this table folowinge I haue not done as in the other statute before, compared my restitucion wyth the faultes crepte into the statute, but oneli haue written that trwe measure, whiche the equitie of the statute doothe pretende. For it were to vyle to iudge of so noble princes and woorthy counsellours as haue auctorised and sette forthe thys statute, that they wolde make one aker in any forme greater then an other, but euery one to bee iuste and equall with eche other, whiche ys the grounde also of my woorke, and hereby maye al menne perceaue howe needefull Arithmetike is vnto the studentes of the lawe. Butte nowe I thynke beste to make an ende of these matters for this presente tyme, sythe the table hath in it none obscuritie, that I shoulde neede to declare.

The bredthe.	The lengthe of the aker.		The bredthe.	The length of the aker.	
perch	perch	feete.	perch	perch	feete.
10	16	0	28	5	11 11/14
11	14	9	29	5	8 11/58
12	13	5 1/2	30	5	5 1/2
13	12	5 1/13	31	5	2 43/62
14	11	7 1/14	32	5	0
15	10	11	33	4	14
16	10	0	34	4	11 11/17
17	9	6 27/34	35	4	9 3/7
18	8	14 2/3	36	4	7 1/3
19	8	6 18/19	37	4	5 18/37
20	8	0	38	4	3 9/19
21	7	10 5/14	39	4	1 9/13
22	7	4 1/2	40	4	0
23	6	15 18/23	41	3	14 73/82
24	6	11	42	3	13 15/42
25	6	6 3/5	43	3	11 77/86
26	6	2 7/13	44	3	10 1/2
27	5	1 5/18	45	3	9 1/6

Schol. In deede sir, I vnderstande the table (as I thinke) by those other whiche you sette forht before. For in the ffrste columne is sette the perches of the bredth of anye a-

ker

ker, and then in the ii. columnes folowynge, appeareth howe manny perches, and howe manny foote the same acre muste haue for his length.

Ma. You take it well: how be it to speake exactly of breadth and lengthe: the fyrste columne dooth some time betoken the breadthe, and sometime the lengthe: for proprelye the longest syde of any square doth lymitte hys length, and the shorter syde dooth betoken the breadth: yet it is no greate abuse in suche tables, where a man can not whell chaunge the tytil, to let the name remayne, although the proportions of the numbres do chaunge, for still by the firste columne is expressed the measure of the one side, and by the twoo other pillers in one columne, is set foorth the measure of the other syde. And this shall bee sufficient now for the vse of the Golde rule. Now sumwhat wil I touche certayne other rules, which for their seuerall names mai seeme dyuers rules and distincte from thys but in deede they are but braunches of it: yet bycause they haue not onlye seuerall workinges in appeerance, but also pleasante in vse, I will geue you a taste of eche of them. As for the rule of fellowship both single and double, with time and withoute time, I shall neede to saye litle more than I haue already saide in teachinge the workes of whole numbres, yet one example or two will we haue to

refresh

refreshe the remembraunce of the same, and to declare certayne propre uses and applications of yt, as thys for one. Foure men gette a bootye or price in tyme of warre, the price is in valewe of mony 8190 li, and bicause the men be not of lyke degree, therefore their shares may not be equall, but the chiefest person will haue of the bootye the thyrde parte, and the tenthe parte ouer: the seconde will haue a quarter and the tenthe part ouer: the thirde will haue the sixte part: and so there is lefte for the fourthe man a verye small portion, but suche is his lot, (whether he be pleased or wroth he muste bee content with one xx. part of the pray. Nowe I demaunde of you, what shall euery manne haue to his share? A questi-on of vn-equal so-ciety.

S. You must be fayne to answere to your owne question, els it is not like to be aunswered at this time.

M. The forme to understande the solution of this question, and all suche lyke, is this: Reduce all the denominators into one numbre by multiplication, excepte that anye of them bee partes of some other of them, for all such partes you may ouerpasse, and take for them all those numbres, whose partes they bee: as in this example the shares bee these $\frac{1}{3}, \frac{1}{10}, \frac{1}{4}, \frac{1}{10}, \frac{1}{6}, \frac{1}{20}$, if I multiplye all the denominatours together, begynninge with 3, & so go on unto 20, it wil make 144000

 but

but considering that 3 is a parte of 6, I shall omit that 3, and likewaies 10 which is apart of 20: I may ouerpasse also, and then is ther but 3 denominatours to multiply, that is 4, 6, and 20, which make 240. whiche summe I take for my worke, bicause all the denominatours will bee found in it. Then I take such partes of it as the question importeth, that is for the firste man $\frac{1}{3}\,\frac{1}{10}$: the $\frac{1}{3}$ is 80: the $\frac{1}{10}$ is 24: whiche I putte in one summe for the first mans share, and it maketh 104. Then for the seconde mans share, I take $\frac{1}{4}$, whiche is 60, and $\frac{1}{10}$ whiche is 24, and that maketh in the whole 84. Now for the thirde man which must haue $\frac{1}{6}$ I take 40. And for the fourth man ther remaineth but 12, whiche is $\frac{1}{20}$ of the whole sum: so that if ye whole pray had been but 240 li, then were the question answered: but bicause the sum was of greater value, by this meanes nowe shall I know the portion of it: I muste set my numbres by the order of the Golden rule, putting in the firste place the numbre that I fond by multiplyinge the denominatours, and in the seconde place the sum of the booty. And loke what proportion is betwene that firste nũber and that second, the same proportion shalbe betwene the partes of the firste number and the partes of the second, comparinge eche to his like: therfore I muste put to the thirde place one of the partes or shares, and then work

The reason of the rule.

th

yͤ former rule of proportion or Golden rule. And bicause I haue 4 seuerall partes of the fyrste numbre, by whiche I woulde fynde out 4 lyke partes of the seconde numbre, therfor muste I make 4 seuerall figures.

S. Nowe I truste I can answere to your question, as by your fauoure I will proue.

a

240 — 8190
104 — 3549

b

240 — 8190
84 — 28661 $\frac{1}{2}$

c

240 — 8190
40 — 1365

d

240 — 8190
85 — 409 $\frac{1}{2}$

And to trie it, I sette the 4 fygures thus, marked with a, b, c, d, to shewe theire ordre. And then in eche of theym I multiply the second numbre be the thirde, and diuide their totall by the first, and so amounteth the fourth, summe whiche I seke for for if I do multipli 8190 by 104, it maketh 851760 whiche being diuided by 240, maketh in the quotient 3549 for the fyrst mans portiō, and so workyng with the other three fygures, I fynde for the seconde man 3866 $\frac{1}{2}$, and for the thirde manne 1365. And then for the fourth manne 409 $\frac{1}{2}$. And so is euerye mannes share sette foorthe in the fygure here answered.

a		b	
240	8190	240	8190
104	3549	84	2866½

c		d	
240	8190	240	8190
40	1365	12	409½

And thus I thinke I haue doone well,

M. If you misdoubt your workinge and lyste to proue it, adde all the shares togither, & yf they make the totall, then semeth it well done.

The prof by addition.

Sch. I may set them thus and then by addition the iuste summe doth amounte, that is 8190: and therefore (as you saye) it semeth to be well wrought.

3549
2866½
1365
409½
8190

But I beseeke you, is there any doubt in this triall, that you vse that word, Semeth?

Ma. You may easily coniecture, that if you did asigne the fyrst mans share to the laste, a so chaunge al the reste that one had an o thers share, yet woulde the Addition apper all one, & therefore is not the profe exact.

The iuste proofe.

But if you will make a iuste proofe, fo the firste mannes parte take $\frac{1}{3}\,\frac{1}{10}$ of th whole summe and if it agree with the num ber in the fygure, then it is well doone. An so de for the second, thirde and fourthe sum mes, and this profe faileth not. Nowe wi

I propounde certayn other questions, whiche haue been set foorthe by certaine learned men, all bee it not without some ouersight, whiche questions I protest heartely I do not repeate to depraue those good men, whose laboures and studies I much praise and greatly delyte in, but onlye accordinge to my profession, to seeke out truth in all thinges, and to remoue all occasions of errore, as muche as in me lieth: and for that cause I will onely name the questions without hurtinge the authors name. The fyrst question is this.

Foure men did buylde a house, whiche cost them 3000 crounes. theyr shares were such, that one man shoulde paye $\frac{1}{2}$ of the summe, & 6 crownes ouer: the second should pay $\frac{1}{2}$, and 12 crounes ouer: the third man must lay out $\frac{2}{2}$, abatinge 8 crownes, and the fourth manne should pay $\frac{1}{4}$. and 20 crounes more. can you answere to this question? A question of buildinge.

S. No in good fayth syr, & that you know best of any man, for I knowe no more then you haue taught me.

M. Than I dare saye you can not doe it, nother yet the beste learned man that euer dyd propose yt, for the question is impossible: for declaration wherof I will be bold to vse first the representation of the numbres in their aptest fourme, (althoughe I haue not yet taught you that manner of woorke) by cause it maye appeare plainely that the question

stion is not possible, for here I haue set the parts, and added them, and they make the whole sum, 1 ¾ and 3 0 more. Now howe is it possyble to diuide truly either gaines eyther charges so, that the particulars shalbe more then the totall?

	½ +	6
	⅓ +	12
	⅔ —	8
	¼ +	20
1	¾ +	30

S. It is against the forme of the profe by addition of the partes.

M. You say truthe. And bicause you shall perceaue it the better, I wyll trye it after the vulgare forme, as in this figure you see, where the ½ with 6 ouer, is 1 5 0 6: for the totall is as you hearde before 3 0 0 0: the ⅓ & 12 more, is 1 9 1 2: the ⅔ would be 2 0 0 0, but then abating 8, it is but 1 9 9 2, & than last of all, the ¼ is 7 5 0, and the 20 more maketh 7 7 0: whiche all beinge added in one summe, do make 5 2 8 0, where the total sum shoulde be but 3 0 0 0. whiche sum if you dyuide by 4/3, so shall you haue ¾ of it, that is 2 2 5 0. and thereto adde 3 0 more, then will those 3 summes make 5 2 8 0: whereby you maye see howe this forme as well as the other, dothe declare that the particulars in that question would make more then the whole summe.

1506
1912
1902
779
6280

300[0]
224[0]
3[0]
528[0]

summe by $\frac{1}{4}$, and 30 more, and therefore can that question not bee accepted as a possible thynge: but yet doo certayne learned menne propound suche questions, and answere to them. Therfore somewhat to saye to their excuse rather of theire good meaninge then for their doinge, I will anone declare what may be saied for theire defence: but in the meane season I will propounde the question as yt maye be wrought by good possibilitie. As yf fower men buyld a house togither, and it coste them 3000, and then for the partition they agree thus: that as often as the fyrste man doothe pay 6 crownes, so often the seconde shal pay 4, the thirde man 8, and the fourthe man 3. Or els thus: that the fyrste man shall pay double so much as the iiij. and the seconde man shall pay $\frac{2}{3}$ of the fyrst mans charge: the thirde man shall paye double so muche as the seconde: (And these twoo wayes are to one ende) but further for their agreement it is apointed also, that the fyrste man shall geue 6 crounes ouerplus, and the seconde 12, and the iiij. shall geue 20, but the thirde man shall geue no ouerplus, butte shall haue 8 crounes abated of his charge. Nowe is the question possible to be soyled, and this is the waye to doo it. Marke the proportion of the seuerall charges, and set out small numbres in that rate, by whiche you may reduce the worke to the Golden rule, as here in the firste forme, ŷ A question of buildinge a house.

numbres

numbres are alreadye named, 6, 4, 8, 3: and in the seconde forme, althoughe they bee not plainly named, yet they may be the same numbres, for 6 is double to 3, and 4 is $\frac{2}{3}$ of 6: and againe 8 is double to 4. Nowe adde these together, and they make 21, whiche 21 muste be set as the fyrste nūbre in the Golden rule, for if it with the ouerplus of eche mannes charge woulde make the totall summe of the charges, then were those seuerall summes, the charges of eche man, beside his ouerplus: but nowe it is not so.

Note. But yet this is true, that looke what proportion eche of these seuerall summes dothe beare to 21, the same proportion dothe the iuste chargis of euery man (beside his ouerplus) beare to the totall of the charges, the ouerplus beinge deducted: wherfore this may you note, that before you do applye the totall of the charges to the Golden rule, you muste deducte the ouerplus which is 6, 12, 120. that is in the whole 38, but then 8 must bee restored for the abatement of the thirde man, and than remaineth to be deducted 30. Take 30 therfore out of 3000, & there wil rest 2970, which I must set in the Golden rule for the second summe: and for the thirde sum I muste put eche of the small numbres beefore mentioned, whiche althoughe they bee not the seuerall charges, yet they represent them in proportion. And so making for euery

man

mans charge a seuerall question, the figures will be 4, which I mark with foure letters, a.b.c.d. thus.

a

21 Z 2970
6 Z 848 $\frac{4}{7}$

b

21 Z 2970
4 Z 565 $\frac{5}{7}$

c

21 Z 2970
6 Z 1131 $\frac{3}{7}$

d

21 Z 2970
3 Z 424 $\frac{2}{7}$

where I haue set for brieffnes the summe of euery mans charge in the fourth place, presupposing that you can tell howe to trye out that fourth summe by so many examples as ye haue had.

Schol. As I truste that I doe vnderstande this forme, so I desire muche to knowe what may be sayd for them that mistooke this question.

M. You seeme so desirous to knowe this errour, that you haue forgotten to examine whether this worke be without errour.

S. Me semeth this worke to be well done, bycause the addition of the iiij. seuerall numbres doth make the totall summe of 2970, whiche was to bee diuided into suche foure partes.

Mayst. But then haue you forgotten that the firste manne muste pay 6 crownes more besides this share, and the seconde man 12

 crownes

crounes more: the third man 8 crounes lesse: and the fourthe man 20 crownes more, for withoute these youre firste totall of 3000 crownes will not be made.

Scholer. Than muste I adde to the firste mannes sum 6 more, and it will bee $854\frac{4}{7}$: and to the seconde sum I must adde 12, and it will be $577\frac{5}{7}$: from the third summe I must abate 8, and then will the sum be $1123\frac{1}{7}$: then addinge vnto the iiij. summe 20, yt will be $444\frac{2}{7}$: and these iiij. summes will make 3000, which is ẏ whole charge.

as in this exãple it may appeare, wher firste I gather ẏ $\frac{14}{7}$, that maketh 12, and so procede I in the addition to the ende.

$854\frac{4}{7}$
$577\frac{5}{7}$
$1123\frac{1}{7}$
$444\frac{2}{7}$
3000

Master. Nowe haue you well done, and this work in the same summes is broughte of other learned men for the true solution of the question, as it was first proponed, which (as I said) was impossible: and nowe examine it by these seuerall summes, and see whether it doe agree with the summes in the question proponed.

The firste man muste paye $\frac{1}{2}$, and 6 ouer of the totall summe, how thinke you, is $854\frac{4}{7}$ the halfe, and 6 more, of 3000?

Scholer. No that ys yt not, for yt would be 1506: and for the seconde man 1012: for the thirde man 1992: and for the fourth manne 770, wherof not one sum agreeth to

thys

this worke: but I meruaile that so wise men could be so much ouer sene.

Mayster. It is commonly sene, that when men will receaue thinges from elder writers, and wyll not examine the thinge, they seeme rather willinge to erre with theyr auncientes for company, then to be bolde to examine their workes or writinges: whiche scrupulositie hath engendred infinite errours in all kyndes of knowledge, and in all ciuile administration, and in euerye kynde of arte. But these learned men did not meane anye other thinge by this question, then to fynde suche numbres as shoulde beare the same proportion together, as those numbres in the question proponed dyd beare one to an other: whiche thynge you shall perceaue more plainly by an other question of theires, that is this.

A man lying on his death bedde, bequeatheth hys goodes (whiche were worthe 3600 crownes) in this sorte. Bycause hys wife was greate with childe, and hee yet vncertaine whether the childe were a male or a female, hee made his bequeste conditionallye, that if his wife bare a daughter, then shoulde the wife haue halfe his goodes, and the daughter $\frac{1}{3}$: but if shee were delyuered of a sonne, than that son shoulde haue $\frac{1}{2}$ of the goodes, and his wyfe but $\frac{1}{3}$. Nowe it chaunced hir to bringe forth bothe a sonne and a daughter, the

A question of a testament.

the question is, Howe shal they parte the goodes agreable to the testatour his will?

Scholer. If somme cunninge lawyers had this matter in scanninge, they wolde determine this testament to be quite void, and so the man to dye vntestate, bycause the testament was made vnsufficient, syth this condition was not expressed in it: and also it might haue chaunced that she shoulde haue brought forthe neyther sonne nor daughter, as often hath ben sene, so is the will vnsufficient in ỹ poynt also.

Mayster. Suche scanners shoulde seeme to cunninge, & yet not so cunninge as cruell, for the minde of the testatour is to be taken fauorably, for the ayde of the legatories, whẽ ther ryseth suche doubtes: But let vs try this worke not by force of law, but by proportion arithmeticall, seinge the testatour dyd mynde to prouide for eche sorte of theim.

Scholer. If the sonne shall haue $\frac{1}{2}$ by force of the testament, so must the mother haue $\frac{1}{3}$. Again bicause she hath a daughter also, therefore ought she to haue $\frac{1}{2}$, and the daughter $\frac{1}{3}$: is bothe waies $\frac{1}{2}$ $\frac{1}{3}$ and $\frac{1}{2}$ $\frac{1}{3}$, whiche commeth to the whole goods, & $\frac{2}{3}$ more. Wherefore it semeth also impossible.

Maister. In thys matter the mynde of the testatour is so to be vnderstand, that such proportion shoulde bee betweene the portion

of

of the wyfe and the sonne, as is betweene $\frac{1}{3}$ and $\frac{1}{2}$, that is, the sonne muste haue $\frac{1}{6}$, for to his mother, so shall he haue 3 to 2, that is as muche as his mother, & halfe as muche more: and the mother muste haue the lyke rate in comparison to her daughter. Then muste I finde out 3 numbres in suche proportion, that the firste maye be as much as the seconde, and halfe as muche more (that is in proportion *sesquialtera*) and the seconde to the thirde in the same proportion, suche numbres be 9, 6, 4.

Scholer. I pray you sir, howe shal I finde out those numbres?

Maister. That will I gladlye tell you: Whatsoeuer the proportion be for any thre numbers, multiply the termes of that proportion togither, & the number that amounteth, shalbe the middle numbre of the 3: then multiply that middle numbre by the lesser terme, and diuide that totall by the greater, and the leaste numbre of the 3 will amounte. So if you multiplie that middle numbre by the greater extreme, and diuide that totall by the lesser extreme, then will the greateste numbre of that progression amount.

To finde thre numbers in any proportion.

Scholer. Then in this example, to finde the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I muste diuyde (as you taught me in dyuision) $\frac{1}{2}$, by $\frac{1}{3}$, and the quotient will be $\frac{3}{2}$, that is $1\frac{1}{2}$, whereby I perceaue that the proportion in this questi-

on

on is as 3 to 2, Therfore (as you taught me euen now) I multiply 3 by 2, and the summe is 6, which must be the middle number: the I multiplye ẏ middle number 6 by 2, whiche is the least terme, and the summe is 12: that doe I diuyde by 3, beynge the greater terme, and the quotient is 4: so is 4 the leaste number of the 3: then I multiplye 6 by 3, wherof commeth 18, and that I diuide by 2, and so haue I 9, whiche is the greatest nũber of the 3.

Ma. An other way yet may you finde the thirde number in anye progression, yf you haue 2 of them. for if the middle number be one of them whiche you haue, thē multiplye it by it selfe (as in this example 6 by 6 maketh 36) and that totall diuide by the other number whiche you haue, and ẏ thirde number will be the quotiente.

Sch. Thē if I diuide 36 (which commeth of 6 multiplied by it selfe) by 4, the quotiente will be 9: and if I diuide 36 by 9, the quotient will be 4. But what if I knowe the firste number and the third, and woulde haue the middle numbre?

M. Multiply the ij. numbers togither, and in their totall you must seke the route of that number, and it shal be the middle numbre: but because as yet youe haue not lerned how to extract routs, therfore vse the firste forme which

whiche I haue taught you, till I teache you to extract routes. And nowe goe forewarde with the answere to the same question.

S. I perceaue then that the sonne muste not haue $\frac{1}{2}$ of the goods, nother the mother $\frac{1}{3}$, nor yet the daughter $\frac{1}{2}$, but yet muste the goodes be diuided in suche proportion, that the sonne shall haue 9 crownes for 6 to hys mother: and ye mother shall haue 6 crounes for euery 4 to the daughter. Then I applye it to the Golden rule in 3 examples thus, where the firste number is the Addition of those thre nũbers, 9, 6, 4, and the thirde is one of them seuerally, the seconde is the totall of the goodes in the testament, & then by the worke of the goldẽ rule, I finde out the fourth number in euery worke, that is for the sonne $1705\frac{5}{19}$ for the mother $1136\frac{16}{19}$, & for the daughter $757\frac{17}{19}$, whiche thre summes added togither doe make the sũme of the whole goodes, as maye be sene by this example.

19 — 3600
9

19 — 3600
6

19 — 3600
4

$1705\frac{5}{19}$
$1136\frac{16}{19}$
$757\frac{17}{19}$
3600

And this (me thinketh) I do perceaue, that bicause in this case ther is a necessary reme diuised agaīst an vrgẽt inconuenience, therefore those lerned mẽ though thei mighte vse the

the like libertie in that other question.

Maister, Your gesse is good, but they hade so good reason for them in the one, as they haue in the other: as in an other example of theyrs it may better appere, that is thys.

An other question of a testament.

A mã lefte vnto his iii. sõnes 7851 crounes to be parted in this sort, that the firste sonne should haue $\frac{1}{2}$, the seconde sonne $\frac{1}{3}$, and the thirde sonne $\frac{1}{4}$, which is not possible, for $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ doth make $\frac{26}{24}$: or $\frac{13}{12}$ that is $1\frac{1}{12}$, so is it more thẽ the whole: but reduce these fractions into one denomination, and they wyll be $\frac{6}{12}$, $\frac{4}{12}$ $\frac{3}{12}$. and so maye you parte the goodes in suche proportion as these 3 numeratours beare togither: that is the fyrste to haue 6 for euery 4 to the seconde: and the seconde to haue 4 as often as the thirde hath 3: and so there porcions will be for the firste $3623\frac{7}{13}$: for the seconde $2415\frac{9}{13}$: and for the thirde $1811\frac{10}{13}$. & those iii. shares added togither will make the totall sũme of the hole goods, as you may easyly see in this example.

$3623\frac{7}{13}$
$2415\frac{9}{13}$
$1811\frac{10}{13}$
7851

An other like questi on.

An other question is there proponed thus: There is 450 crounes to be diuided betwene 3 men, so that the first man must haue $\frac{1}{2}$ $\frac{1}{3}$, the seconde man $\frac{1}{3}$ $\frac{1}{4}$, the thyrde man shall haue $\frac{1}{4}$ $\frac{1}{5}$.

Scholer. I meruaile that any man shold be so ouersene to propounde that question

on as a thinge possible , sith $\frac{1}{2}\frac{1}{3}\frac{1}{3}\frac{1}{4},\frac{1}{4}\frac{1}{5}$, dooe make $1\frac{13}{15}$, that is almoste double the whole summe.

But I perceaue it mighte bee thus proponed , that as often as the fyrste man dyd receaue 5 0 crounes, so often the secōd manne should receiue 3 5 ,& the thirde man 2 7, for $\frac{1}{2}\frac{1}{3}$, is equall to $\frac{50}{60}$: and so is $\frac{1}{3}\frac{1}{4}$ equall to $\frac{55}{60}$, and $\frac{1}{4}\frac{1}{5}$ is $\frac{27}{60}$, and so workinge the question, the 3 fygures wyl appeare in thys forme: whereby the firste mans portion is founde to bee $200\frac{50}{56}$: ye secōd mans part is $140\frac{35}{56}$: ye third mans share is $108\frac{27}{56}$: which in the whol doth make 45 0 crounes, yt was the whole sum to be diuided betwene them.

112	450
50	$200\frac{50}{56}$
112	450
35	$140\frac{55}{56}$
112	450
27	$108\frac{27}{50}$

Ma. And thus you are (I thynke) sufficiently instructed in the rule of felowship.

The Rule of Alligation.

Now wil I go in hand with the rule of Alligation, which hathe his name for that by it there ar diuerse parcels of sundrye pryces, and sundrye quantities alligate, bonde, or mixed together, wherby also it might be wel called the rule of Myxture, and it hathe greate vse in composition

The rule of Mixtures.

tions

sion of medicines, & also in mixtures of mettalls, & som vse it hath in mixtures of wines, but I wishe it were lesse vsed therin than yt is nowe a daies. The ordre of the rule is thys: When anye summes are proponed to bee mixed, set them in ordre one ouer an other, and the common numbre whereunto you wyll reduce them, set on the left hand, then marke what summes bee lesser then that common numbre, and which bee greater, and with a draught of your pen euermore linke .ij. numbres togyther, so that one bee lesser then the common numbre, and the other greater then he, for twoo greater or twoo smaller can not well be lynked together, and the reason is this, that one greater and one smaller maye be so mixed, that they will make the meane or common numbre verye well: but ij. lesser canne neuer make so manye as the common numbre being taken ordrely: noe more can twoo summes greater then the meane neuer make the meane in due ordre, as it shall appeare better to you hereafter. And as it is of necessitye to linke euery smaller ones (at the leaste) with one greater, and euerye greater with one smaller, soo it is at libertye to lynke them oftener then ones, and so maye there bee to one question manye solutions. When you haue so lynked them, then marke howe muche eche of the lesser numbres ys smaller then the meane or common numbre

A questiō of mixing of wyne.

bre

bre, and that difference set againste the greater numbres, which be linked with those smaller, eche with his matche still on the ryghte hande: and likewaies the excesse of the greater numbers aboue the meane, you shall sette before the lesser noumbers whiche bee combined with theim. Than shall you by addition bringe all these differences into one summe, which shall bee the firste number in the Golden rule: and the seconde number shall bee the hole masse that you will haue of al those particulars: the thirde sum shall bee eché difference by it selfe, and than by theim shall bee founde the fourthe noumber, declarynge the iust proportion of euery particular in that mixtur. As now by this examples I wil make it playne.

There is iiij. sortes of wyne of seuerall prices, one of 6 d a galon, and other of 8 d, the thirde of 11 d, and the fourth of 15 pes the galon, of all these wynes woulde I haue a mixture made to the summe of fiftye gallons, and so that the price of eche gallon maye bee nyne pens. Nowe demaunde I howe muche must bee taken of euerye sorte of wyne? The reason of this rules

Scholar. If it shall please you to worke the firste example, that I maye marke the applyenge of yt to the rule, then I truste I shall bee able not onelye to doe the like, but also to see reason in the order of the worke.

Master. Marke then this forme, and the placinge of euerye kynde of noumbre in yt.

The prices seuerall.		The differences.	
	6	6	a
9	8	2	b
	11	1	c
	15	3	d
		12	

a

$$\begin{array}{ccc} 12 & & 50 \\ 6 & & 25 \end{array}$$

b

$$\begin{array}{ccc} 12 & & 50 \\ 2 & & 8\tfrac{2}{6} \end{array}$$

c

$$\begin{array}{ccc} 12 & & 50 \\ 1 & & 4\tfrac{1}{6} \end{array}$$

d

$$\begin{array}{ccc} 12 & & 50 \\ 3 & & 12\tfrac{3}{6} \end{array}$$

Here (you see) I haue set downe the seueral prices which be 6, 8, 11, 15, & haue linked together 6 with 15, & 8 wt 11: ye common price 9 I haue set on the lefte side, & the difference betweene it, and euerye particular price I haue set on the right hand, not against the summe whose dyfference it is, but againste the summe that it is lynked withall, so the difference of 15 aboue 9, is 6, which I haue set not against 15, but against 6 that is linked with 15, and the difference betweene 6 and 9 (that is 3) I haue set againste 15. So lykewaies the difference betweene 8 and 9 is but 1, that haue I set against 11, and the difference of 11 aboue 9 (which is 2) I haue set against 8. Then adde I all those 4 differences, and they make 12, whiche I set for the fyrste numbre in the Golden rule: the second numbre I make 50, whiche is the sum of

of the galons that I woulde haue, and the thirde summe is euery particular difference. Nowe if you worke by the golden rule, you shall finde the number of galons that shall be taken of eche sorte of wyne: for the better distinction wherof, I haue set these letters, a, b, c, d, bothe against the numbres for which the workes be serue, and ouer the workes also whiche seuerally serue, for eche of them. And now if you liste to examine the truth of these workes, adde those iiii. summes togither and they wyll make 50, that is the totall which I woulde haue, as be this example you may easilye perceaue. And for to proue howe the pryces do agree, do this: Multiplye this totall summe 50 by the common price 9, and it will make 450: than keepe that summe by it selfe, and afterwarde multiplye euery seuerall summe of Galons, by the price belonginge to the same galons, and if that sum doe agree with this, whiche you haue kept fyrst, then is your worke well done. As here, 25 is the numbre of Galons of 6 pence, multiply then 25 by 6, and it maketh 150, whiche you shall set downe: then multiply $8\frac{2}{6}$ by 8, whiche is the price for that numbre of galons, and it wyll make $66\frac{4}{6}$: so againe

The profe of this rule.

25
$8\frac{2}{6}$
$4\frac{1}{6}$
$12\frac{3}{6}$
50

150
$66\frac{4}{6}$
$45\frac{5}{6}$
$187\frac{3}{6}$
450

againe 4 1/6, multipled by 11, doothe make 45 5/6. And laste of all 12 3/6 multiplyed by 15 maketh 187 3/6, And these added togyther, dooth make 450, as in the example annexed you may see: wherfore seynge it dooth agree with the former summe of 50 multiplied by 9, I maye iustely affyrme this worke to bee good and wel doone.

The vari ation of this question.

And nowe to prowe howe you can doo the like, I propounde the same question, onelye willinge you to vse some other forme of combininge or linking the summes

Scholer. That shall I proue wyth youre fauoure, and therfore I combine 8 with 15, and 6 with 11, and then the fourme wyll be thus.

whereby amounteh the same summe intotall of the differences, as dyd before: and yet nowe ye differences be altered, as the cōbination ys chaunged, whereof I vnderstande the reson by your former worke. And therefore here appereth no straunge thinge, but that nowe I must haue 8 2/6 galons of 6 d, and 25 galons of 8 d. and 12 galons and

$\frac{3}{6}$ of 11 ℔, and so consequently 4 gallons and $\frac{1}{6}$ of 15 ℔, so that multiplying 8 $\frac{2}{6}$ by 6, it maketh 50, & then 25 multiplyed by 8, maketh 200: lykewaies 12 $\frac{3}{6}$ multiplyed by 11, yeldeth 137 $\frac{3}{6}$: and 4 $\frac{1}{6}$ multiplyed by 15, maketh 62 $\frac{3}{6}$, whyche iiij. summes added in one, wyl yelde in the totall 450, whiche agreeth with the Multiplication of 50 (beynge the totall summe of Gallons) by 9 the common or meane pryce.

$$\begin{array}{r} 50 \\ 200 \\ 137\frac{3}{6} \\ 62\frac{3}{6} \\ \hline 450 \end{array}$$

Maister. Seinge you conceaue this worke so well, I will propounde an other example vnto you of more varietye in the alligations or combinynges: As thus.

A marchaunte beynge mynded to make a bargaine for spyces in a myxte masse, that is to say, of cloues, nutmegges, saffron, pepper, ginger, and almoundes: the cloues beinge at 6 ß a pounde, the nutmegges at 8 ß, saffron at 10 ß, pepper at 3 ß, Ginger at 2 ß, and Almondes at 1 ß. A questiō of spices.

Now woulde hee haue of eche sorte some, to the valewe of 300 li. in ẏ whole, & eche pounde one with an other to beare in pryce 5 ß, howe muche shall hee haue of eche sorte?

Scholar. That wyll I trye thus.

Fyrste I sette downe those syxe seuerall prices, and at the left hande

1	5	a	a: 18 — 300	5 — $83\frac{1}{3}$	
2	1	b	b: 18 — 300	1 — $16\frac{2}{3}$	
3	3	c	c: 18 — 300	3 — 50	
6	3	d	d: 18 — 300	3 — 50	
8	2	e	e: 18 — 300	2 — $33\frac{1}{3}$	
10	4	f	f: 18 — 300	4 — $66\frac{2}{3}$	
	18				

I set the common prise 5.s. Then I linke them thus, 1 with 10, 2 with 6, & 3 with 8.

M. I had minded to haue combined them in more varietie, but I am contente to see youre owne worke first, and then more varieties in combination may folowe anone.

Scho. Then to continewe as I beganne, I seke the difference betweene 1 and 5, whyche is 4, and that I set againste 10: then againste 1 I set 5, whiche is the excesse of 10 aboue 5: so I gather the difference betwene 2 and 5, which is 3, and that I set againste 6, bicause it is combined with 2: and likewaies the difference of 6 aboue 5 (which is 1) I set againste 2. Then take I the difference of 3 from 5, which is 2: and that I set agaynste 8, and before that 3 I set the difference of 8 aboue 5, which is 3. Then gather I al these differences by Addition, and they make 18, which I sette for my firste numbre in the golden rule, and so appeareth by those wor-

kes

ke, that of almoundes I muste take 8 3 li $\frac{1}{3}$, of ginger 1 6 li $\frac{2}{3}$, pepper 5 0 li, of cloues, 5 0 pounde, of Nutmegges 3 3 pounde $\frac{1}{3}$ of Saffron 6 6 pound $\frac{2}{3}$

Then for tryall hereof, I multippye euery parcelle by hys seuerall pryce, as 8 3 $\frac{1}{3}$, whyche is the sum of almondes I multiplye by 1, which is their price.	8 3 $\frac{1}{3}$ 3 3 $\frac{1}{3}$ 1 5 0 3 0 0 2 6 6 $\frac{2}{3}$ 6 6 6 $\frac{2}{3}$ 1 5 0 0

Also 1 6 $\frac{2}{3}$ the summe of ginger, I multiple by 2, whiche is the price of it. And so eche other in this kynde, as this table annexed doothe represent: and then adding them altogither, I fynde the totall to be 1 5 0 0, whiche also wil amount by the multiplication of the grosse masse of 3 0 0 by the common pryce 5, wherfore it appeareth wel wrought.

Mayster. Nowe wyll I make the alligation to proue your cunning some what better, but because you shall not thinke youre selfe pressed to muche, I wyll also note the differences, as in this example you maye se,

5	1	1 3	4
	2	3 5	8
	3	5	5
	6	4	4
	8	4 3	7
	10	3 2	5
			3 3

3 3 — 3 0 0
4 — 3 6 $\frac{4}{11}$

3 3 — 3 0 0
4 — 3 6 $\frac{4}{11}$

3 3 — 3 0 0
8 — 2 7 $\frac{8}{11}$

3 3 — 3 0 0
7 — 9 3 $\frac{1}{11}$

3 3 — 3 0 0
5 — 4 5 $\frac{5}{11}$

3 3 — 3 0 0
5 — 4 5 $\frac{5}{11}$

where I haue alligate 1 with 6 and 8, and
therfore haue I set against 1, both their dif-
ferences, that is 1 and 3. Likewaies bicause
2 is combined with 8 & 10, I set before him
their differences, 3 and 5. Against 3 I haue
set only 5, which is ẏ difference of 10, with
whome thre is combined only: likewaies 6 is
only alligate to 1. and herfore ys the diffe-
rence of 1 onely set against it: 8 is linked w
1 and 2, & therfore hath he against him there
both differences 3 & 3, & 10 is ioined wͭ 2 & 3,
therfore hath he their differences 4 & 2. And
bicause of ease for you, in an other columne
I haue set the differences reduced into one
numbre for euery seuerall sorte, and haue al-
so added theim togither, whereby appereth
that they make 33. And so consequently you
see the workes of the golden rule set forth
for the sixte seuerall drudges: I haue not ad-
ded letters, a, b, c, &c. as before, bycause I
woulde not wishe you to cleaue styll to these
elementary aydes, but accustome memorye
to truste to hir selfe, so shall occasion of neg-
ligēce be best auoided. And as for the proofe
trye it at a more laysure, bicause the tyme
now is shorte, & you sufficiently instructed in
that profe.

And there resteth diuers thinges behynde
yet, of which I wold gladly gyue you some
taste before our departure.

Schla

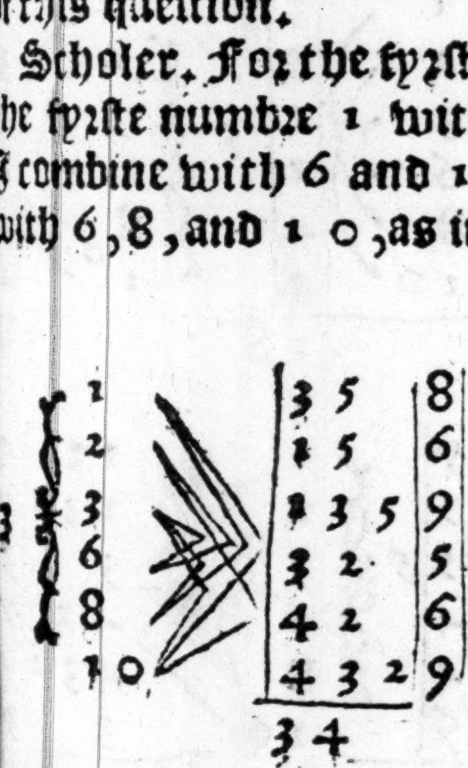
Scholer. But if it maye please you to lette me see all the variations of this question before you go from it, for me thinketh I coulde varie it ij or three waies more yet.

Master. I am contente to see you make two or thre variations, but I woulde bee loothe to staie to see all the variations: for it maye be varyed aboue 300 waies, al thoughe many of them woulde not well serue to this purposse.

Scholer, I thought it impossible to make so many variations.

Master. Meruaile not ther at, for as some questions of this rule maye be varied aboue 1000 waies: but I woulde haue you forget suche fantasies, tyll a tyme of more leysure. And nowe go forwarde with some variation of this question.

Scholer. For the fyrste variation, I lynke the fyrste numbre 1 with 8 and 10, and 2 I combine with 6 and 10, then ioyne I 3 with 6, 8, and 10, as in this forme.



43 Z 300   43 Z 300
8 Z 56 $\frac{35}{43}$ 5 Z 34 $\frac{37}{14}$

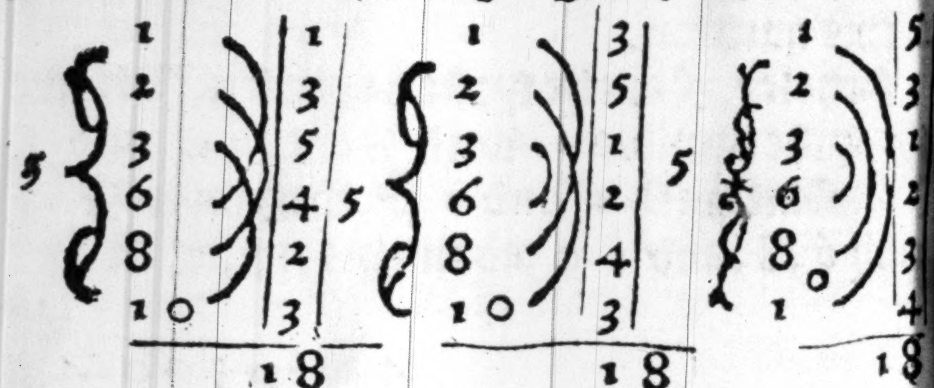
43 Z 300   41 Z 30
6 Z 41 $\frac{47}{41}$ 6 Z 62 $\frac{34}{43}$

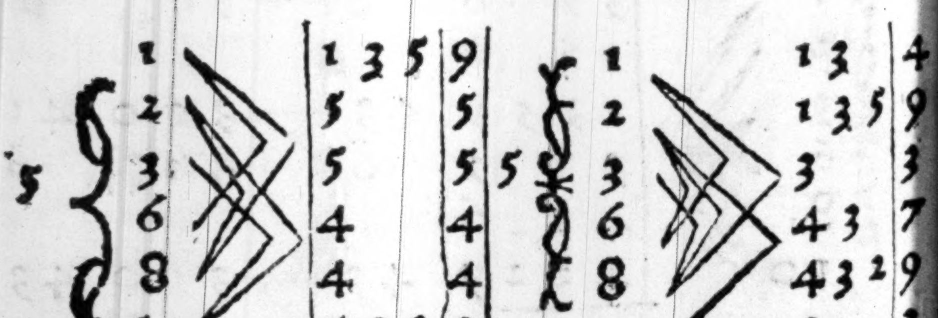
43 Z 300   43 Z 300
9 Z 41 $\frac{34}{41}$ 9 Z 66 $\frac{34}{41}$

And so dothe there appeare the portion of weight for euerye kinde of drugge in thys myxture. Now for the triall.

Mast. Nay staye there. you shall not neede to make triall in one example so often: or yf you liste to dooe it by your selfe, I am content. But nowe set forthe (for declaracyon that you conceaue the rule) two or three examples of seuerall combinations, and then will we passe to som other example, and so ende this rule.

Scholer. As it pleaseth you so will I doe. And these bee the varieties in which as the combinations are seueral, so doth it plainely appere, that the differences by whiche the proportion of eche seuerall kynde is taken, are also seuerall. And yet I se in the iii. firste





of these v. varieties, and in one other before the totall summe of the differences to be one, that is to say 18, wherby I perceaue that ẏ varietie of their mixture dothe depende of the varietie of their differences seuerall, & not of the varietie of theire totall summe.

Mast. So is it. And seyng you conceaue it so wel, I wil make an ende of this rule, only exhibitinge to you one question or twoo of the mixture of metals, that by it you maye deuise other like, and exercise your selfe therin also, bicause the vse of it serueth often in businesse of charge, not so muche for goldesmithes, as for coynage in myntes. First I demaunde of you this question: If a mynte maister haue golde of 22 karectes, and some of 23 karectes, some of 24: Againe some of 15, some of 16, and som of 18 karectes, & woulde myxe them so, that hee mighte haue 100 vnces of 20 karectes, howe much shall he take of euery sorte? A question of mixing of Golde.

S. To know that, I set the numbres in order thus.

20	15	2
	16	3
	18	4
	22	5
	23	4
	24	2
		20

20	100		20	100	
2	10		5	25	
20	100		20	100	
3	15		4	20	
20	100		20	100	
4	20		2	10	

Maister.

M. You haue wroughte the question well, but how chaunced you made no doubte of that newe name Karecte?

Sc. Bicause I thoughte it oute of tyme to demaunde suche questions nowe, seinge you make so muche haste to ende: and againe in this case the proportion of the numbres is sufficiente for my purpose in thys worke, trustinge that an other tyme you wyll instructe me as well of this, as of sundrye other thinges, whiche I haue hearde you to talke of, so I haue a great desire to knowe them.

M. Your aunswere is reasonable: and your request and trust (with gods helpe) I intende to satisfy. And now to go forwarde with this matter, let me see youre examination of thys laste worke.

S. First for the one part, I adde	10
together al the particular summes	15
as they appeare in the worke, and	20
they make 1 0 0, as here by their	25
addition it doth appeare.	20
	10
And so it seemeth that the summes	100

150	are well gathered: but for the far-
240	ther trial of them, I multiply first
360	2 0, which is the commō or meane
550	sum of the karectes, by 1 0 0, whi-
460	che is the sum of the whole masse
240	which I would haue, & it maketh
2000	2000. Then I multiply euery par-

ticular

ticular summe by the karectes that yt dothe cōtain, as 10 by 15, and that maketh 150. Likewyse I multiply 15 by 16, & it yeldeth 240: so 23 by 18 maketh 360. And 25 by 22 yeldeth 550: lyke waies 20 by 23 bringeth forth 460. & last of all, 10 multyplyed by 24, yeldeth 240: whiche summes al ioyned together make 2000, that doth agree with the like summe before. wherfore I may wel say that the worke is good. And now if it please you I wold set forth som varieties of this question, to proue my wytte.

Maister. Go to, let me see.

Scholar. Here be foure varieties.

20	15	3 4	7
	16	3	3
	18	2	2
	22	2	2
	23	5 4	9
	24	5	5
			28

20	15	2 3	5
	16	3 4	7
	18	4	4
	22	5	5
	23	5 4	9
	24	4 2	6
			36

20	15	2 3 4	9
	16	4	4
	18	3	3
	22	5	5
	23	5 2	7
	24	5 4	9
			37

And more yet I coulde make, but not like to the number that you spake of in ẏ variation of the other question.

Maister.

20	15	4	4
	16	4	4
	18	2 3 4	9
	22	2	2
	23	2	2
	24	5 4 2	1 1
			3 2

Mayst. That wil I teache you at more leasure, seynge it is a thinge rather of pleasure; then of anye necessitye.

But nowe for your exercise in this rule, one other question I will propose. A mynte maister hath 6 ingottes of siluer of sundrye finenesse, some of 4 ounces fine, and some of 5 vnces, some of 6; and other of 8, some of 1 1, & other of 1 2: and hys desire is to mixe 5 0 0 pound waight, so that in ye hole masse euery pounde waighte shoulde beare 9 vnces of fine syluer, howe muche shall hee take (saye you) of euery sorte of syluer?

A questiō of mixing of siluer.

Sc. To finde out that, I set the numbres thus in ordre.

9	4	2	
	5	2	
	6	3	
	8	3	
	1 1	5 4 9	
	1 2	3 1 4	
		2 3	

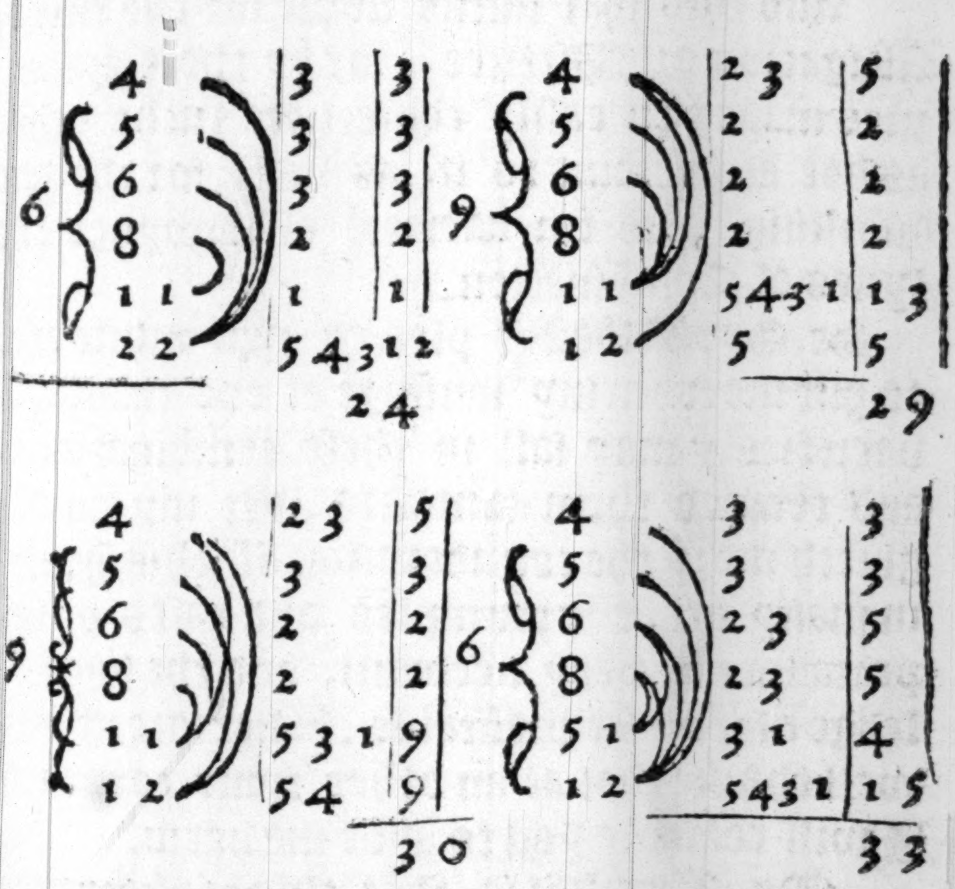
And gatheringe the differēces, it will appeare, that of the fyrste sorte ther must bee 43 $\frac{11}{23}$: of the seconde lyke muche: of the third sort, 6 5 $\frac{5}{23}$: and of the fourthe sort as muche. of the fifte sorte 1 9 5 $\frac{15}{23}$: and of ye sixt sort 8 6 $\frac{22}{23}$ which in the whol will make 5 0 0 li waight; and in vnces 4 5 0 0, ye is of the fyrste sorte 1 7 3 $\frac{21}{23}$; and of the second

sort

sorte $217\frac{2}{23}$: of the third sorte $391\frac{7}{23}$: of ye fourth sort $521\frac{17}{23}$: of the fift sorte $2152\frac{4}{23}$, and of the syxte sorte $1043\frac{11}{23}$, which all togither doe make 5400 unces, agreable to ye multiplication of 9 by 500.

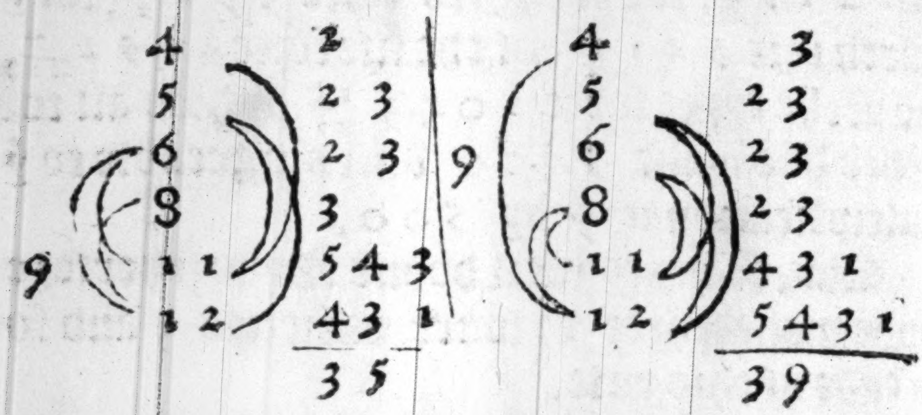
Ma. This is well doone of you, therfore nowe make three or foure varieties, and so an ende of this rule.

Sch. These 4 varieties I set for example



Master. And by these it appereth that you can finde out more, with whiche I wyll not now medle, saue only for to shew you an easy helpe in drawinge the lines of Combination, I wyll set forth twoe varieties here.

 And



And this shal suffise nowe for the rule of Alligation or Mixture, for by these exaumples maye you easily coniecture suche other as doe appertaine to it, as well for the due workinge, as for varietie of drawynge the lynes of Combination.

Sc. Syr, Albeit it pleased you while ere, to put me from my musinge at the manyfold varieties, ẏ may fall in these combinations, and termed them fantasies, yet my fantasie giueth me, ẏ the consideration of this shoulde in many other exaumples and cases of importaunce by very needefull, and the knowe-ledge of it moste profitable. Therfore ye may well thinke that at an other tyme conuenient I will requeste you to ayde me herin.

Ma. Truth it is, that this consideration may fall in practise as well politike as philosophicall, and sundry waies in them bee applied, therfore whan time shall fall feete for the discussinge of this consideration, you shall not want my helpinge hande.

The

NOw will I briefly also teache you somewhat of the rule of Falsehode, whiche beareth his name, not for that it teacheth any fraude or falsehode, but for that by false numbres taken at al aduentures, it teacheth how to finde those true numbres that you seeke for.

The occasion of the name.

S. So might any other rule be called the rule of Falshode, for they worke by wronge numbers, and by them find out the righte numbers, so doth the rule of alligation, the rule of Fellowship, and the Golden rule, partlye.

Maister. In the Golden rule, the rule of Fellowship, and the rule of Alligation, although the numbers that you worke by, be not the true numbers that you seeke for, yet are they numbres in iuste proportion, and are found by orderly work, where as in this rule, the numbers are not taken in any proportion, nor found by orderly worke, but taken at all aduentures.

And therfore I somtimes beinge merye with my friendes, & talking of such questions haue caused them ye proponed such questions, to call vnto them such children or ydeotes, as happened to be in the place, and to take their answere, declaring yt I woulde make them to solue those questions that semed so doubtfull.

And in deede I did answere to the questio

and

and worke the triall thereof also, by those answeres whiche they happened at all aduentures to make: whiche numbres seynge they be taken as manifeste false, therefore is this rule called the rule of False positions, and for briefnesse, the rule Falshode, whiche rule for redinesse of remembraunce, I haue comprised in these fewe verses folowing, in forme of an obscure Riddle.

Gesse at this worke as happe doth leade,
By chaunce to truth you may procede.
And first worke by the question,
Although no truthe therin be don
Suche falsehode is so good a grounde,
That truth by it will soone be founde.
From many bate to many mo,
From to few take to fewe also.
With to muche ioyne to few againe,
To to few adde to many plaine.
In crossewaies multiply contrary kinde,
All truth by falsehode for to finde.

The sence of these verses, and the sum of this rule, is this.

When anye question is proponed appertayning to this rule, firste imagine any number that you lyste, whiche you shall name the Fyrst position, and put it in steede of the true number,

number, and then worke with it as the question importeth: and if you haue missed, then is the laste number of that worke other to greate or to little, that shall you note as hereafter shall be taught you, and you shall call it the firste errour.

Then beginne againe, and take an other numbre, whiche shalbe called the seconde position, and work by the question: if you haue missed againe, note the excesse or default as it is, and cal that the Second errour. Then multiplye crosse wayes the firste position by the seconde erroure, and againe the seconde position by the firste errour, and note theire totalles seuerally by the names of Totalles. Then marke whether the two errours were bothe lyke, that is to saye, bothe to muche, or both to litle: or whether they be vnlike, that is the one to muche, and the other to little, for if they be like, then shall you subtracte the one totall from the other (I mean the lesser from the greater) and the remainer shall be your diuidende. So muste you abate the lesser errour out of the greater, and the residue shall be the diuisour. Nowe diuide the diuidende by that diuisour, and the quotient will shewe you the true numbre that you seeke for. But and if the erroures be vnlike, then muste you adde bothe those totalles (which you noted) together, and take that whole number for the diuidende: so shall you

adde bothe the errours togither, and that whole numbre shall be the diuisor. And the quotient of that diuision shal geue you the true numbre that the question seeketh for: And this is the whole rule.

S. This rule semeth so vnlike any other, that withoute sum example I shall not easily vnderstande it.

A Question of masondry

M. Therefore take this example: A mason was bounde to builde a wal in 40 dayes, and it was couenanted so with him, that euery daye that he wroghte he shoulde haue for his wages 2 ß, 1 peny, and euery daye that he wroghte not, he shoulde bee amersed 2 shyllinges, 6 pence. so that when the wall was made, and the rekenninge taken of the daies that he wroughte, and of the other that hee wroughte not, the mason hadde cleerelye but 5 ß, 5 pence for his woorke. Now doe I demaunde, how many dayes did he woorke of those 40, and how many did he not worke?

Sch. I praye you expresse the order of the worke, that I maye partly by imitacion, and partly by comparinge it with the rule, be able agayne to do the like.

M. This order shall you kepe in the work of this rule, firste take sum number as you list at aduenture, as for example, I saye hee plaied 12 daies, & wrought 18 daies. Now cast you the wages of euerye daye & se whether it will agree with the summe of 5 ß, 5 pence

3 pence.

S. The 28 daies that he wrought, after 25 pence by the day, yeldeth 700 pence. The the 12 daies that he wrought not, at 30 pēs eche day, doth amount to 360 pence, whiche if I abate out of 700, ther resteth 340; but you say he had not so muche.

Mast. He had but 65 pence, and by this suppositiō he should haue had 340, therfore is this sum to muche by 275, which summe I muste sette downe after this sorte as you see here, where I haue made a crosse (commonly called saincte Andrewes crosse) and at the ouer corner on the lefte hande, I haue set the first position 12, and at the other corner vnder it, I haue set 275 which is the firste erroure, with this figure +, whiche betokeneth to muche, as this line —, plaine without a crosse lyne, betokeneth to little. On the righte hande of the crosse I haue left two like rowmes for the seconde position and his errour. Therefore to prosecute the worke, I suppose he playd 16 daies, and wrought 24.

12 ╳

275 +

Sc. I was a while in doubte why you named the daies of his workinge, seeyng they be not set in the figure: and I doubted howe you knewe them, or els whether that you did suppose them at all aduentures, as you dyd

the daies that he played: but nowe I gather, that seinge 40 daies is the whole tyme limited, then the daies that he plaied beinge supposed, the rest of 40 must nedes be the daies that he wrought, and therfore 28 folowed 12 of necessitie, and 24 foloweth 16 also of necessitie: but yet I scarse perceiue why you set not in the figure as well 28 as 12.

Maist. It forceth not which of theym I take, so that in the seconde position I take the numbres of the same nature, that is here both of working daies, or bothe ydle daies: but now examine you this second position.

Sch. If he playde 16 dayes, then abatynge 16 times 30 d, the summ wil be 480 d. And for 24 daies that he wrought, euerye day yeldinge 25 d, the totall is 600 d: so that abatynge 480 out of 600, there resteth 120, & as you say it should be but 65, therefore it is to much by 55, that must be set on the right hande of the figure at the nether parte, and ouer it on the same side 16, which is the seconde position, thus.

12	16
2754	554

And as I gather by youre wordes, it were all one if I did set 28 in steede of 12, & 24 in stede of 16.

M. So were it. But this shall you marke, that of what nature so euer the twoo positions be: of the same nature is the Quotient. Therefore when the positions in this questi- are

are 12 and 16, which bothe being numbres of the playinge daies, the quotient shall declare the true number of the playinge daies, where as if the positions had bene 28 and 24, which are supposed to bee the workinge daies, then woulde the quotient declare the true number of the workinge daies, and not of playinge days. as it wil do now. And therefore to continue the worke of this question, and to fynde the true noumbre of plainge daies, I must multiply crossewaies the firste position 12 by 55, that is the second errour, and the totall will be 660. then I multiply 275 by 16, and it yeldeth 4400. Now bycause the errours are lyke, that is to saye, bothe to muche I must subtracte 660 out of 4400, and so remaineth 3740, which is ÿ diuidend. Againe I muste subtratte the lesser errour 55 out of 275, that is the greater errour, and there will remaine 220, whiche shall be the diuisour. then dyuiding 374, by 220, the quotient will bee 17. Wherefore I say nowe constantlye, that 17 is the true number of dayes that the mason plaiede: and then it foloweth that he wrought 23 daies, and so is the question answered.

Now for the order of tryall of this woorke, there nedeth none other profe but only this to work with this number accordinge to the question, and if it agree, then appeareth the number to be it that you would haue. As here now

The profe of this rule.

now seinge he wrought 23 daies, & must haue for euery daie 25 pens, ỹ whole sum cōmeth to 575. Thē again, seing he plaied 17 daies & must abate 30 pence for euery day, ỹ whole sum of ỹ abatement will be 510. therefore I subtract 510 out of 575, & ther wil remaine 65, which maketh 5 s. 5 d, ỹ clere wages of ỹ mason for his work, according to the questiō.

S. Nowe I trust I vnderstande the worke and the rule so well (and the better by this proofe) that I can bee able to doe the lyke. And for a proofe I take the same question all saue the laste numbre, where I will suppose that he had 10 s, for his wages clere. And now to gesse at the numbre of ỹ daies that he wrought, I suppose first that he wrought 20 dais: then say I, if he wrought 20 daies, hys wages must be 500 d, then did he play other 20 daies, for whiche must be abated 600 d, and then hee leeseth 100 d. And so am I at a stay, for it is not lyke to your former worke.

M. You shoulde haue required of me some question, and not haue taken a question of your owne fantasying, vntyll you were more expert in this arte: for so might yeu as well happen on an impossible question as on a possible: but now to go forwarde, considre that this nūbre is to little by 220, seing he shuld gain by your supposition 120, and in this positiō he leseeh 100, those both make 220 which you shal set down for the firste erroure

with

with this signe —, betokeninge to little, as heere in this forme folowinge dothe appeare. And now for the rest go forward your self ones againe.

20

229—

S. As my errour hath vttered my folly, so it hath procured me better vnderstandinge. Now therefore consideringe this position not to solue the question, I take an other, supposinge that he wrought 30 daies, thē for his wages he must be alowed 750 d, & for the 10 daies which he wrought not, he must abate 300 d, & so remaineth cleere 450 d, but it shoulde be onlye 120 d, therfore it is to much by 330, which I set down in ẏ figure with the former position, & his errour & the figure appeareth thus.

20 30

220— 330+

Nowe muste I multiply in crosse waies 220 by 30, and it wil be 6600. Thē againe I multiply 330 by 20, and it will be also 6600. Wherfore if I shall subtract the one out of ẏ other, there wil remaine nothing to be the diuidend.

M. In this you forget youre selfe agayne: for in as muche as the signes of the erroures be vnlike, therfore muste you worke by Addition, addyng together those two totals to make the dyuidend, and addinge also the two erroures to make the diuisoure. And bicause you shal no more forget this part of the rule, take

take this brife remembrance.

Vnlike require Addition,

And like desire Subtraction.

Sch. You meane that if the erroures haue lyke signes, then muste the diuidende and the diuisor be made by subtraction, as is taughte before: and if those signes be vnlyke (as in this laste example they bee) then muste I by addition gather the diuidende and the diuysor. Therfore must I ad 6600 to 6600, & it wil be 13200, which shalbe ẏ diuidend. Thē againe, I ad 220 to 330, & it wil be 550, whiche must be the diuisoure: wherefore diuiding 13200 by 550, ẏ quotient will be 24, wherby I knowe ẏ the mason wrought 24 daies: and then it foloweth that he plaied 16 daies.

M. Examine your worke whether it be agreable to the question or no.

Sc. For 24 daies worke, the wages muste be 600 d, and for 16 daies whiche ẏ mason wrought not, there muste bee abated 480 d, and then remaineth clere to the mason 120 pence, as the question importeth: wherefore it is euident, that 24 is the true nūbre of the daies that he wrought.

M. Although you seeme now to vnderstande this worke, yet to acquaint your minde ẏ better with the newe trade of this rule, I thinke it good to propone to you fyue or sixe examples more, before I make and ende of it.

Scholer.

S. Sir I thanke you, that you doe so consider my commoditie & profite in knowledge, for undoubtedly it is practyse & exercise that maketh men prompte and experte in euerye kynde of knowledge.

Maister. You say well, so that they folowe some certaine preceptes to gouerne and rule theire practise by, els may practise procure custome of errour, and a repugnance to exactnesse of knowledge, namely as longe as the erroure is not plainly knowē to the vulgar sorte. But to returne to our worke. Ther is a seruante that hath bought of veluet and damaske for his maister 40 yardes, the veluet at 20 s a yard, and the damaske at 12 s, & when he commeth home, his master demaūdeth of him, howe muche he hath boughte of eche sort: I can not tel (saith he) exactly, but this I know, that I paid for damaske 48 s more then I paid for veluet. nowe muste you gesse how many yardes ther is of eche sorte. A questiō of wares.

S. Althoug h the gesse semeth difficulte, yet I wyl proue what I can do: for I remembre your saying, ỹ it forceth not howe fonde or false the gesse be, so it be somwhat to the question, & not an answere of a contrary matter.

Therefore firste I Imagine that hee bought 20 yardes of damaske, for whyche hee shoulde paye after the former pryce 240 shillinge, then muste he nedes haue of veluet other 20 yardes (to make vp the 40 yardes)

des (and that would cost 400 s. So that the totall of the price of the damaske is lesse the the summ payde for veluet 160 s, and should be more by 48, therfore the firste erroure is 208 to lyttle. Then begynne I agayne, & suppose hee bought of damaske 30 yeardes, that coste 360 s, then had he but 10 yeardes of veluet, whiche cost 200 s, and nowe the price of damaske is greater then the pryce of the veluet by 160 shillinges, and should bee but 48, therfore is the seconde erroure 112 to muche, whiche I sette in fourme of a fygure, as heere doothe appeare. Thenne doo I multiplye in crosse waies 208 by 30, and the summe will bee 6240. Also I multiply 112 by 20, and there wil amount 2240. And in as muche as the signes of the erroures bee vnlyke, I knowe I muste worke by Addition, therfore adde I those two totales togither, and thei make 8480, which is the diuidende then adde I also the twoe errours togyther 208 and 120, & they make 320, whyche is the diuisoure. Wherfore diuidinge 8480 by 320, the Quotient wil bee $26\frac{1}{2}$, whiche is the true summe of yeardes of damaske that he bought: and in veluet 13 yardes $\frac{1}{2}$, and that appeareth by examination, thus: $26\frac{1}{2}$ yeardes of damaske at 12 s. the yearde, maketh

20	8 −	39
30	8 +	1124

maketh 318 ß: then in veluet he had but 13 yeardes and $\frac{1}{2}$, that coste 270 ß, at 20 ß the yearde. Now subtract 270 out of 318 & ther will remaine 48, which is ỳ nūbre of shillings ỳ the damaske did coste more then the veluet.

Master. Now shall you haue a question of an other kynde.

There are thre men that doo owe monie to me, and I haue forgotten what the totall summe is, and what the particulars be. — A questiō of debt.

S. Whi? thē it is impossible to know ỳ debt.

M. Peace, ye are to hastye: there is more helpe in it then you yet see, I haue thre seuerall notes, whereby it appearethe that I did confer their debtes togither. founde ỳ the debt & the seconde to amount to 47 li, ỳ debte of the firste man & the thirde did make 71 li, & the seconde man his debte wͭ the third, did rise to 88 li. Now, can you tell what eueri man did owe, & what was the whole totall?

Sc. Naye in good faith: but as I percaue ỳ it muste be foūd by cōiecture, so will I gesse at it, supposinge that ỳ first mā did owe 20 li, & the seconde man 30, and the thirde

Mast. Nay stay there, for you ar to farre gone al ready, you may not suppose a seuerall summe for euery man, for it is inoughe to suppose one summe for the firste man, and let the other rise as the question importethe. Therefore seynge you set the firste man his debt to bee 20 li, the seconde mā can not owe

30 li, for the declaration is that their debets added togyther, did make 47 li, so muste the seconde man his debt be but 27 li. Now this seconde debt with ẏ thirde must make 88 therfore subtract 27 out of 88, & ther wil remain 61, as the thirde man his debet. Then saythe the declaration, that the firste & thirde menns debts do make 71: but by this suppositiō thei make 81, that is 10 to muche: which I must set for the firste errour. Nowe worke you the seconde position.

S. I suppose the firste mannes debt to be 24 pounde, then must the seconde mans debt (by your declaratiō) be but 23 li. seyng bothe they do make but 47 li. Also the second man his debte with the third, do make 88 li, and the seconde man oweth but 23, therefore the thirde man must ow 65 li. Nowe the thirde, manns debte with the first, sholde mak by the declaration 71 li, and they doo make 89 li, that is 18 li to muche: and that is the second erroure, whiche I set downe with the firste, and their position in this forme, and then do I multiply in crosewaies 20 by 18, & it is 360. Also 10 by 24 maketh 240. And

20	X	24
10		18

bycause the signes of the errours be lyke, I muste worke by subtraction, therfore I subtract 240 out of 360, & ther resteth 12[0] whiche is the diuidend: then do I subtract

1[2]

16 out of 18 by the same reason, and so is the diuisour 8, whiche is founde 15 tymes in 120: therfore I say ẏ the firste man did owe 15 li, and then the seconde man must owe 32 li, for those two do make 47 li: & the thirde mā his det is 56, for so much remaineth if I bate 15 out of 71, or if I take 32 out of 88

M. For the third example take this easye question for the varietie in worke. Two men hauing seuerall sūmes whiche I knowe not, doe thus talke togither: the firste saithe to the seconde, If you giue me 2 ℔ of your money, then shall I haue 3 times so muche monye as you, the seconde answereth: It were more reson, that our sūmes were made equall, and so wil it be, if you giue me 3 ℔ of your moneye. Nowe gesse what eche of them had. The third question.

S. I imagine that the firste had 9 ℔.

M. Cōsider euermore in your ymaginationis ẏ you take a likly sūme, as in this question take suche a sūme that hauing 2 added vnto it, may be diuided into 3 partes euen. Note.

S. Why? I remember you sayde before, it forced not howe fondely so euer I gessed.

M. As for the possibilitie of the solution it is truth, but for easinesse in worke, the aptest numbers are most conuenient.

S. I thought no lesse, and therfore I toke 9 as an apt number to be parted into 3, but I perceaue I shoulde haue considered ẏ aptnesse of that particion after the Addition of

2 vnto it, and then 7 had ben more meeter.

Ma. That is truthe, and then shulde the seconde mā his sū be 5: for although he haue now but ꝧ third part of 9, that is 3, yet you must remember that he lent the first man 2, and so had he 5.

S. Then to go forward: if the second man had 3 of the first man, then shold he haue 8, and ꝧ first man but 4, so hath he double to the first man: yet he sayde in the question, they shoulde haue equall. wherfore it appeareth ꝧ he hath 4 to much. Therfore I note ꝧ error with his suppositiō, & gesse againe that he hath 10 ß. whereunto I adde two shillinge borowed of the second man, and then hath hee 12 shillinges, so the seconde man hathe remayninge but fower, whereunto if I ad the two that he lent to the firste manne, so hadde hee but 6 ß at the beginninge. Then take 3 shyllinges from the first manne, and giue to the seconde, then hath the firste man but 7, and the second hathe 9, whiche are not equall, but there are 2 to manny: wherfore I sette downe bothe the positions with their errours as here you see, and multiply acrosse,

7	10
4+	2+

so commeth their 40 and 14: and bicause the signes bee like, I take 14 out of 40, and so resteth 26 to be the diuidend. the likewaies I take 2 out of 4, and there resteth 2,

by

by which I diuide 26, and the quotient wyl be 13, whiche is the summe that the first mā had. And so appereth that two beynge added therto, the summe will be 15, so hath the seconde man nowe but 5, & before he had 7: the take thre fro the firste and put to his seuen, so haue eche of them ten, and that is equall, as the question would.

M. For the fourth exāple take this question. One man sayde to an other: I thynke you had this yeare two thousande lambes, so hadde I sayde the other: but what w paying the tythe of them, and then three seuerall losses, they are muche abated: for at one tyme I loste halfe as many as I haue now left: and at an other time the thirde part of so manye: and the thirde time $\frac{1}{4}$ so manye. Nowe gesse you how manye are lefte.

The fourth exāple.

Sc. Bicause here is mention made of certaine parts, I must take a nūber yͭ may haue al those partes: yͭ is to say, $\frac{1}{2}$, $\frac{1}{3}$ & $\frac{1}{4}$, whiche will bee 24, howe bee it 12 hath the same partes. Therfore firste I take 12 to be yͤ nūber yͭ do remaine, so hath he lost 6, 4 & 3, yͭ is 13: & in yͤ whole 25, but it shoulde be 2000.

M. Ye are deceiued yet stil, you haue forgotten the $\frac{1}{10}$ part, which muste be defalked, that is 200, so there remaineth but 1800: and nowe go on againe.

Sch. Then to finde the erreur, I take 25 oute of 1800, & there remaineth 1775 to

fewe, which I set for the first erroure. Then for the second position I take 24, whose halfe is 12, ÿ third part 8, & the quarter 6, wher by riseth 50, whiche is to little by 1750, therfore I set downe both the positiõs with their errours thus.

12	24
1775—	1750—

And multiply in crosse waies 1775 by 24, wherof cõmeth 42600 Also I multiplye 1750 by 12, & there reseth 21000. And bicause ÿ signes are like, I do subtract the one from the other, and so remaineth the diuidend 21600: then do I subtract 175 out of 17750, & ther resteth 25: by which I diuide 21600, & the quotient is 864, wherof ÿ halfe is 432, & the third parte is 288, the quarter is 216, which all beinge added together wil make 1800. And yf you adde thereto the tenthe which was abated before, then wil the whole sum be 2000.

864
432
288
216
1900

And nowe dothe there come a question to my memory whiche was demaunded of mee, but I was not able to answere to it, and nowe me thinketh I could solue it.

Maister. Propone your question.

Sch. There is supposed a law made, that (for furtheringe of tillage) euerye man that doth keepe sheepe, shall for euery 100 sheepe are and sowe one acer of grounde; and for

his

his allowance in sheepe pasture, there is appointed for every 4 sheepe 1 acre of pasture: Nowe is there a riche sheepemayster whiche hath 7000 acres of ground, & wold gladly kepe as many sheepe as he might by ỹ statute, I demaunde howe many sheepe shall hee kepe?

M. Answere to the question your selfe.

Scho. Firste I suppose he may keepe 500 sheepe, and for them hee shall haue in pasture after the rate of 4 sheepe to an acre, 125 acres, and in earable grounde 50 acres, that is 175 in all: but this erroure is to little by 6825. Therfore I gesse againe that he may keepe 1000 sheepe, that is in pasture 250 acres: and in tillage 100 acres, which maketh 350, that is to little by 6650.

500 1000

6825— 6650—

These bothe errours with their positions I set downe as you see, and multiplye in crosse 6825 by 1000, & it maketh 6825000. Thē I multiplye 6650 by 500, & it doth amounte to 3325000, which sum I do subtract out of the fyrste: & there remaineth 3500000 as the diuidend. Also I dooe subtract the lesser erroure out of the greater, and so remayneth 175, by which I diuide the saide diuydend, and the quotient will be 20000, so ỹ I se that by this rate hee that hath 7000 acres of grounde, may kepe 20000 sheepe: &

therby I coniecture that manye men maye kepe so many sheepe, for many men (as the common talke is) haue so many acres of grounde.

M. That talke is not likely, for so muche grounde is in compasse aboue 48 $\frac{3}{4}$ miles: but leaue this talke, and returne to youre questiōs least your pointinge be scarse well taken.

S. In deede I remembre ỷ the Egyptians did grudge so much against Shepherdes, tyl at length they smarted for it, & yet thei wer but smal shepmaisters to som men that bee nowe, and the shepe are waxē so fierce now & so mighty, ỷ none can withstande them but the Lyon.

An other way of workinge.

M. I perceaue you talke as you heare some other: but to the worke of your question: both this laste question and the nexte before mighte be wrought without the seconde position, by the rule of proportion, as this. Whan in this question ye found in the first erroure, that for 500 Shepe there must be 175 acres, then might you reduce it to the Golden rule, thus: If 175 acres will admit in allowance 500 sheepe, thā 7000 wil haue 20000. And so

7000		500
175		20000

by one position with the helpe of the Golden rule, may you answer the question. Likewaies for the question of lambes, when you hadde found that 12 came of 25, you might haue set the figure thus as you see, and haue

1800		12
25		864

haue saide: If 25 do leaue but 12, what shall 1800 leaue? & it would appeare to be 864.

Sch. Sir I thanke you for this ayde, for it doth much shorten the worke of this rule.

M. Yet againe I will shewe you an other way to answer to this laste question without this rule of false position, & that by the rule of felowshippe: for it appeareth in the proponing of the question, that ten sheepe muste haue in pasture 2 acres and $\frac{1}{2}$, and for theym must ther be eared but one acre: so it foloweth that for two acres eared, there muste bee fiue set to pasture. And if you put them both into one summ, they wil make 7. Therfore looke what proportion 7 beinge this totall, doothe beare to 5 & to 2, suche proportion shall anye total in this question beare to the pasture grounde and the eared grounde. An other way yet.

S. This serueth wonderous aptly. Therfore to proue it, I demaunde this by the former supposition: If a man haue 300 acres, how muche shall hee leaue in pasture, and howe muche shall torne to tillage? You saye that as 7 is to 5, so shall 300 bee to ỹ acres of pasture: & as 7 is to 2, so is 300 to the acres of tyllage, wherof for bothe I haue sette examples here folowing,

wherby appeareth ỹ of pasture there shall bee $214\frac{2}{7}$ acres, and of tillage $85\frac{5}{7}$, which both summes added together, do make 300.

	7	5
300		$214\frac{2}{7}$
	7	2
300		$85\frac{5}{7}$

 Maister.

An other question.

M. Now take an other example: A man hathe three siluer cuppes with one couer, the couer waieth 18 ounces, ỹ second cup waieth euen halfe the waight of the firste & the third. Now if the couer be put to the first cup, they way iust as much as all the three cuppes doe way: and the couer be ioyned with the seconde cup, they way as muche as the seconde twyse and the third: & if the couer be put to the third cup, they will way twise as much as the firste and the seconde cup. Now trye you what was the iust waight of euery cuppe.

S. I doe set the waight of the firste cuppe to be 9 ounces, then in as muche as these two (that is to say the couer and the first cup) doe wey the waight of the thre cups, I see that ỹ thre cups must weye 27 vnces, for so muche is 18 and 9. Also bicause the first & the third do wey double so much as the seconde, therefore is it the thirde part of that waighte ỹ is 9 and then woulde it folowe, that the thirde cup also shoulde wey 9 ounces: but then ỹ questiō sayth, that the couer beinge ioyned to the second cuppe, they wey as much as the seconde twise & the third ones, that should be 27, & so it dothe: then beynge ioyned with the thirde cuppe, they shoulde wey twise as much as the firste and the second, that should be 36, and they wey but 27, so is that erroure 9 to litle. Then begin I againe, and say, that the fyrste cuppe doth weye 12 ounces, whiche I

the I ioyne with the couer, and they make 30 vnces: then seinge the seconde is $\frac{1}{3}$ of that waight, it must needes wey 10 ounces, and the thirde muste awaey 8 ounces, seeing the firste and thirde muste wey 20. Nowe put I the couer to the seconde cuppe, and they weye 28 ounces, whiche shoulde be euen so: then ioyne I the couer with the third cuppe, and so shoulde it weye twise the firste and the second, that is 44 ounces, and they doe weye but 26, that is 18 to lyttle. those errours with their positiōs I set down & multiply in crossewaies 9 by 12, wherof cōmeth 108. Also 9 by 18, and that yeldeth 162. & in as muche as the signes bee like, I abate the lesser out of the greater, and there dothe remain 54. Then do I also abate the lesser errour from the greater, and so remaineth 9, by which I diuide 54, and the quotiente is 6. whiche I take for the true weighte of the firste cuppe: which being ioyned with the couer, muste wey as muche as the three cuppes, so do they wey but 24 ounces. Then seynge the seconde cuppe is the thyrde parte of that weight, for the other two cuppes (you say) must wey double his weight, the weighte of the second cup is 8 ounces, & so the weight of the thirde must be 10 ounces. Now put the couer to the second cup, and it wyll make 26

9 12

9— 18—

ounces: that muste bee the weighte of the second twise, and the thirde ones, that is twise 8, and ones 10: and so is it. Again put the couer to the third cuppe of 10 vnces, and they must weye twyse as muche as the fyrste and the seconde, that is 28: and so is all agreable.

Master. Then answere to this question.

A questiō of water

There is a cesterne with iiij. cockes, containinge 72 barrels of water. And if the greatest cocke bee opened, the water will auoide cleane in vj houres: at the seconde cocke it will aske 8 houres: at the thirde cocke it will auoyde in no lesse then 9 houres: and at the smallest it wyll require 12 houres: Now I demaund, in what space will it auoide all the coks being set open?

Sch. Fyrst I imagine that it wil auoyde in 2 howers.

M. Then must there auoide by the first cock $\frac{1}{3}$ of the water, that is 24 barrels, & by the second cocke $\frac{1}{4}$, that is 18, and by the third cock $\frac{2}{9}$, that is 16 barrels, & by the smallest cocke $\frac{1}{6}$ that is 12 barrels, al which summes put together, doo make 70, as by their addition it doth appeare: but it shoulde be 72, therfore the errour is 2 to fewe.

Scholer. Then I begynne againe by your fauour, bicause I thinke I vnderstande the worke, & put three howers for the due tyme, so shall there run

$$\begin{array}{r} 24 \\ 18 \\ 16 \\ 12 \\ \hline 70 \end{array}$$

out

out at the greatest cock $\frac{1}{2}$, ŷ is 36 barrels, & at the seconde hole $\frac{3}{8}$, that is 27, and at the third cock $\frac{1}{3}$, ŷ is 24, & at the smallest hole $\frac{1}{4}$, that is 18 barrels, whiche altogether dooe make 105, & shuld be but 72, so is it to much by 33, therfore do I set the errors in ordre of ŷ figure, wyth their positiōs, & work by multiplication in crosse, saying: 2 times 3, is 6, & 2 times 33 maketh 66, & bycause the signes are vnlike, I muste adde those two totals together, which make 72: also I adde ŷ ij. errours, & they make 35, by whiche I diuide 72, & the quotiēt riseth $2\frac{2}{35}$, wherby I see, ŷ all the cockes beinge set open, the water wil auoid in 2 hours & $\frac{2}{35}$ of an hour.

2 — 3

2 — 33 +

Mast. This exercise maketh you to growe expert in the rule. Therfore I wil inure you somwhat more with a question or twoo.

There were two men that had been parteners, & had in accompt betweene them 003 ducketes: wherof the one should haue for his part 180, & the other 120, but in ŷ parting of them they fell at variance, so that eache of them catched as many as he coulde: yet afterward beinge reconciled, they agreed that hee which had gotten moste parte of them, sholde lay downe $\frac{1}{4}$ of them againe: and he that hadde gotten least, shold lay down $\frac{1}{3}$ of those whiche he had taken, and then partinge them into two equal parts, ech man to haue half therof, A questiō of parteners.

and

and so had they theire iuste portions as they ought. now I demaunde of you what eche of them had gotten by the scamblinge?

Sc. I suppose he ỷ had leaste, gat 108 ducates, then the other had 192, wherfore in laying downe againe of the 192 there was putte doune $\frac{3}{4}$, that is 144. and so had he left but 48. Also of ỷ 108 ther was laied doune 36, that is $\frac{1}{3}$, and so he had left 72. Then I put together 144, and 36, & it maketh 180 which I parte into two partes euen, and so commethe 90 to be giuen to eche of them: which sum put to 72, maketh 162: & ioyne to 48, it maketh 138: & now I doubt how I shall goe forward.

Note. M. You nede not to take but one of them, whiche you like, the greater or the smaller, for all commeth to one purpose: and so maye you compare it that you take to anye of the other summes, remembringe that you make comparison to the some in the seconde worke. as for example of the firste parte: If you compare 138 with the lesser summe due, that is 120, so is it 18 to much: and if you compare it with the greater summe, than is it 42 to little. Agayne, if you compare 162 to the greater summe, the erroure will be 18, as it was in the other: but it wyll haue a contrarye signe, and if you compare it with the lesser summe, it will be 42 to much: so that the errour both waies is either 18 or 42. &

as

as for ÿ signes it little forceth, for in the is no thinge considered here, but likenesse & vnlikenesse, whiche in this case doth neither further nor hinder. But nowe go on with the worke.

Sc. If it be so, then am I out of my greatest doubte. Then I ioyne that 90 (which I founde as the halfe of the latter partitiō) vnto 48, which is lefte with the one man, & so hath he 138, whiche (I may saye) is 18 to many for the least shold be but 120: that errour dyd I note, and then make a new position, supposing the one man to haue 204, & the other to haue 96, wherfore of the 204 there muste be laid down 153, & so remaineth with him 51 Also of the 96 ther must be laid downe $\frac{1}{3}$ ÿ, is 32, & so resteth with that man 64. Nowe of the 153 & 32 I make one sum as 185, which I muste diuide into 2 equall partes, & so eche man shall haue 92$\frac{1}{2}$, whereunto if I adde their former portions reserued, then ÿ one shall haue 156$\frac{1}{2}$, & the other hath 145$\frac{1}{2}$ Wherefore I take the lesser summe nowe againe as I dyd before, that is 143$\frac{1}{2}$. & finde that he hath to many by 23$\frac{1}{2}$, for he shoulde haue but 120, so haue I for my twoo positions ij. errours, whice I sette downe as here maye be seene, eche erroure vnder his position, and then by the rule I doe multiply in crosse waies 108 by 23$\frac{1}{2}$, and theyr

108	96
18+	23$\frac{1}{2}$+

A questiō of shepe & tyllage.

seth

riseth 2538 which I note: the again I multiply 96 by 18, & therof amounteth 1728 Now bicause the signes are bothe like, that is both to many. I muste worke by subtraction, & so abating 1728 out of 2538 ther will rest for the diuidend 810: then for the diuisour I subtract 18 out of $23\frac{1}{2}$, & there remaineth $5\frac{1}{2}$, by whiche I diuide 810, & the quotient will be $147\frac{3}{11}$, which is ye iust portion of him that had the leaste sum. And if I do subtract it out of 300 beyng the totall sum, then will there remaine $152\frac{8}{11}$, as the portion that the other did get.

M. For the proofe of this worke, you may chose whether you wyll examine those numbers according to the forme of the question, or els worke by other 2 positions for to find ye second nuber, and if those positions bringe the same nubers that did amount by the firste 2 positios, the doth eche worke cofirme other.

Sch. By your pacience, I will proue bothe waies: not onely to see their agrement, but also to accustome my mynde to those workes: for I perceaue it is exercise that muste be the chief engrauer of these rules in my memorye

Ma. You consider it well: then goo to.

Sch. Fyrst I will by two other positions try to find the portio of him whiche had most

M. Although you may do it with anye positions, yet to se the agrement of your worke the better, take the same positions that you

dyd before, comparinge them now to ye grea-
ter (as you did before) vnto the lesser.

Scho. Then I suppose that he that had most, had 192, so had the other 108. Now if I take $\frac{3}{4}$ of 192, that will be 144, & there will rest to that mā but 48, and from ye second, whiche had 10, if I toke $\frac{1}{2}$, ye is 36, there wil remain to him 72: thē ioinig 144 with 36 it will make 180, ye halfe whereof beinge 90 If I. adde to eche of those twoo mens portions remaining wt them, the one shal haue 138, & ye other 162, of whiche two I take ye greater (that is 162) and se it to be 18 to few, for it shold be 180, ye error I note vnder his positiō. Then for the second position I take (as I did before) 204 for ye one, and so resteth 96 for the other: thē take I $\frac{3}{4}$ of 204 and it wil be 153, & there resteth to him 51. Also of the 96 I take $\frac{1}{3}$ that is 32, & there remaineth to him 64. Nowe put I that 32 vnto 153, & it yeldeth 185 which beynge parted in equall values, maketh 92 $\frac{1}{2}$, to be added to eche mans remainder, and so the one hath 143 $\frac{1}{2}$, & the other 156 $\frac{1}{2}$. wherfore I take the greatest som, and it is 23 $\frac{1}{2}$ to lyttle, that doe I note also, and set both these errors vnder their positions, as in this exāple dothe appeare.

192	X	204
		23 $\frac{1}{2}$—

And then multipliing 192 by 23 $\frac{1}{2}$, ther doth arise 4512,28—

Againe

Againe, I multiply 204 by 18, & it maketh 3672 which I do subtract out of 4512, bycause the signes by lyk, & there resteth 840 for the diuidend: then subtracting 18 out of $23\frac{1}{2}$, there wil remaine $5\frac{1}{2}$, whiche I must take for the diuisor. And so diuidinge 840 by $5\frac{1}{2}$, ye quotient will bee $152\frac{4}{11}$, wherby I haue founde an agreable sum, to the whiche I founde by ye former positions, for him yt had moste, which if I do subtracte out of 300 yt is the total, ther will rest $147\frac{7}{11}$, which was the portion of him that hade the laste parte.

M. So by diuers positions you see, yt one doth confirme ye worke of ye other. Nowe examine those two nūbres by ye forme of the questiō, & so shall you proue your worke good also

S. If that he which gate most had $152\frac{4}{11}$ then muste hee laye downe $\frac{3}{4}$ of this summe, that is $114\frac{3}{11}$, & so shall remaine with hym but only $38\frac{1}{11}$. The other whiche had least that is $147\frac{7}{11}$, muste putte downe of his sum $\frac{1}{3}$, that is $49\frac{2}{11}$, & so doth there remaine with him yet $98\frac{5}{11}$. Then doo I adde togyther $114\frac{3}{11}$, and $49\frac{2}{11}$ and it will mak $163\frac{5}{11}$, which I muste parte into ij. equall partes, and that wil be $81\frac{8}{11}$, to be giuen to eche of them: so puttinge $81\frac{8}{11}$ vnto $38\frac{1}{11}$, there doth amounte 120 iuste, whiche is the true portion of him that shoulde haue the lesser summe, and adding $81\frac{8}{11}$ to $98\frac{5}{11}$, the total will bee 180, the true portion of the o-

ther

ther. And so is the worke by this prooke also tried to be good. And this I marke by the waye, that in their scamblinge, he gatte most (as it chaunceth oftē) that ought to haue had least by iust partition.

M. Let your studi be to learne truth and iust arte of proportion, and to distribute and part accordinge therunto, as often as occasiō shalbe minnistred. And here wold I make an ende of this rule, saue that I remember one pleasaunt question which I cannot ouerpasse, which I will declare somewhat largely, bicause you shal as wel understande some reason in ẏ pleasaunte inuention, as apte procedinge in the wittie workinge therof.

Hiero kynge of the Syracusians in Sicilia, hadde caused to bee made a croune of goulde of a wonderfull weyght, to be offered for his good successe in warres: in makynge wherof, ẏ goldsmyth fraudulently toke out a certayne portion of golde, and put in syluer for it, so that there was nothinge abated of the ful waight, although there was much of the value diminished: whyche thinge at length beynge uttered (as no euill can allwayes lye hidde) the kynge was sore moued, and beynge desirous to try the truthe withoute breaking of the croune, proponed the doubte to Archimedes, unto whose wytte no thinge semed unpossible. which althoughe presently he could not answere unto, yet he

An exam ple of mixture of Golde and siluer

had good hope to deuise som policy for that in uetion. And so musinge theron, as hee chanced to enter into a baine full of water to washe him, hee obserued that as his bodye entred in to the baine, the water did runn ouer the tub: wherby hys ready wit of suche small effectes coniecturinge greater workes, conceiued by and by a reason of solution to ẏ kings questiō, & therefore reioycing excedingly more then yf he had gotten the croun it selfe, forgat that he was naked, & so ran home, crying as hee ran, ἕυρηκα, ἕυρηκα, I haue founde, I haue found. And therevpon caused ij. massy peeces, one of gold, and an other of syluer to bee prepared, of the same weight that the saide croune was of: and consideringe that golde is heauier of nature then syluer, and therfore golde of lyke weight with syluer, must needds occupie lesse roume, by reason it is more compacte and sounde in substance: hee was assured that puttinge the masse of golde into a vessell brymme full of water, there woulde not so muche water runne ouer, as when he shoulde put in the syluer masse of the like weighte. Wherefore he tried bothe, and noted not onelye the quantities of the water at eche tyme, but also the difference or excesse of the one aboue the other, wherby hee learned what proportion in quantitie is betweene golde and syluer of equall weyght. And then puttinge the croune it selfe into the vessell of water brimefull (as

before marked how much water did runne out then, and comparing it with the water ÿ ran out when the golde was put in, noted howe much it did exceede that; and likewaies comparinge it to the water ÿ ran out for the syluer, marked howe much it was lesse then that, and by those proportions found out the iust quantitie of golde that was taken out of ÿ croune, & how much syluer was put in steede of it. But seinge Vitruuius which writeth this historie, doothe not declare the particular woorke of thys triall, it shall be noe inconuenience to suppose an exaumple for declarations sake, wherin although the true and iuste proportions be not expressed, yet the fourme of triall shall bee truely set foorthe. And for an example I suppose the weight of the croune to be 8 li, and so of eche of the other twoo masses. And when the masse of golde was putte into the water, I imagine that there ranne oute 2 pounde of water; and when the masse of silUer was put in, I suppose there ranne oute 3 pound $\frac{1}{2}$. Agayne when the croune was put in, there ran out 2 pounde $\frac{1}{4}$. Now to knowe what quantitie of syluer was in the croun, worke by the rule of false position, and imagyn that there was 2 pou̅d of syluer, then must ther be 6 pound of golde. The̅ say thus by the rule of Proportio̅, If 8 pound of golde do expel 2 li. of water, what shal 6 li. expell? and it will be 1 pound $\frac{1}{2}$. Againe for the syl-

 uer

ner: If 8 li. of sylluer expell 3 li. $\frac{1}{2}$ of water, what shal 2 li of syluer put out ? it will be $\frac{7}{8}$. Nowe adde those ij. weightes of water togyther, & thei wil make 2 li $\frac{3}{8}$, & it shoulde bee by the supposition 2 li. $\frac{1}{4}$, so is it to much by $\frac{1}{8}$.

S. Nowe do I understande the worke as I thinke, therfore I pray you let me worke the rest of the question. And bicause this fyrste supposition did erre, I note that position & his errour, and take a newe position, esteeming the syluer to be but one pound, so must there be in gold 7 pound. Then saye I: If 8 li. of gold yelde 2 li. of water, what shall 7 li yeld? & it will be 1 li. $\frac{3}{4}$. Againe if 8 li. of siluer expell 3 li $\frac{1}{2}$ of water, what shal 1 li. expell? & it wil be $\frac{7}{16}$. Nowe must I adde those twoo summes togither. & thei make 2 li, $\frac{3}{16}$ & thei shoulde make 2 li $\frac{1}{4}$, so ys yt to little by $\frac{1}{16}$. Therfore I set the positions with their erroures in order, as heere foloweth. And the I multiply in crosse waies 2 by $\frac{1}{16}$, and it maketh $\frac{1}{8}$: likewaies 1 multiplied by $\frac{1}{8}$ maketh $\frac{1}{8}$, and bicause the signes be unlike, I must adde those ij summes, whiche make $\frac{1}{4}$ and that is the diuidend. Againe I must adde $\frac{1}{8}$ to $\frac{1}{16}$, and it wil be $\frac{3}{16}$, that is the diuisour. Nowe I shall diuide $\frac{1}{4}$ by $\frac{3}{16}$, and the quotient wil be $\frac{16}{12}$, that is 1 $\frac{1}{3}$ wherby I knowe that there was put 1 li. &, $\frac{1}{3}$ of syluer into the crowne,

$$\begin{array}{ccc} 2 & & 1 \\ & \times & \\ \frac{1}{8}+ & & \frac{1}{16}- \end{array}$$

croune, & so muche golde taken out for it.

M. Proue it nowe by examinacion accor- dinge to the question.

S. If there were 1 pounde $\frac{1}{3}$ of siluer, than was there of golde 6 pound $\frac{2}{3}$. Nowe saye I by the rule of proporti- on: of eight pound of gold expel two poūd of water, what shall 6 poūd $\frac{2}{3}$ expel?

$$\begin{array}{ccc} 8 & & 2 \\ & Z & \\ 6\frac{2}{3} & & 1\frac{2}{3} \end{array}$$

It wyl be one pounde $\frac{2}{3}$.

$$\begin{array}{ccc} 8 & & 3\frac{1}{2} \\ & Z & \\ 1\frac{1}{3} & & \frac{7}{12} \end{array}$$

Againe if 8 li of syluer ex- pell 3 li $\frac{1}{2}$ of water, what shall 1 $\frac{1}{3}$ expel? It will be $\frac{7}{12}$. Nowe must I adde tohither 1 li $\frac{1}{3}$ & $\frac{7}{12}$ & thei wil make 2 li, $\frac{3}{12}$, that is 2 li $\frac{1}{4}$, accor- ding to ẏ supposition of the question: whereby I perceaue ẏ worke to be wel done. And as I can not but muche reioyce of this excellēt in- uentiō, so my desire is kinled vehemētly to be perfectli instucted in euery part therof, & namly in this point, whether ẏ proportiō be- twene water & gold be suche, ẏ for 8 li of gold put into a vessel ful of water, there shall run oute ij li. of water. And for as muche syluer, whether 3 li. $\frac{1}{2}$ of water woulde auoyde.

M. I perceaue your meaning, and coniec- ture your imagination to bee thus: that yf you knewe the exacte proportion betweene golde and syluer, and water, bothe in theire waight and in ther quantities, then coulde

Bb.iii. you

you easily fynde out the mixtures of them. whiche thinge I haue reserued for an other worke that intreateth suche matters specyally. And at this time you must consider, ÿ you learne Arithmetike, which intreateth of the manner to solue doubtfull questiōs touthing number, wout regarde what matter is signified by that nūber, els were it necessary in Aritmeticke to teache al artes, seinge in it may be moued questiōs of al artes. But seing you ar so desirous to knowe this thinge, I will tell it you in suche a sorte, ÿ you shall practise your art in finding it, & propounde it in forme of a question. Golde beareth greater proportion to water then siluer doth, & their two proportions be in proportiō togither as $\frac{48}{25}$. But to helpe you sōewhat in this riddle, you shall note ÿ the proportiō of quick siluer vnto water is ÿ iust middle nūber proportionall in Progression Geometricall, betweene the proportions of golde & siluer vnto water. And his proportiō is, as $\frac{290}{21}$. Nowe if you will knowe ÿ iust nūbers of these 3 proportions, then must you finde out 3 numbers in progression geometricall, wherof ÿ middle moste must be $\frac{290}{21}$, & the firste muste be vnto ÿ last, as 25 to 48. And thus I will leaue you to find those nūbers when you be at leasure.

A questiō of the proportion of gole siluer and quicke siluer vnto water.

S. Yet sir I thāke you hartily for this muche for now I se the possibility to fynd thē out. Howbeit bicause this questiō semet strāge if it

yf it might please you to instructe mee somewhat in the order workinge for it, I shoulde the more easily finde the true working.

Ma. You desire to muche ease if you wyll study for nothing: therfore to occasion you to studye the better, I will leaue this doubte wholy to your own search. But as touchinge the generalty of the rule, Archimedes neded not to take twoo masses of golde and siluer equall in waight with the crown, for the proportion might as well bee founde in any other weight, yea although the masse of Gold were of one waight, and the masse of syluer of an other. As for example. If the crowne were of 8 pound weight, as I did suppose, and I haue not so much other fine golde, but onelye 1 li. and tryng that by water, and findinge that it dothe expell but $\frac{3}{4}$ of an vnce of water, yet than by it may I infer, that 8 pound of golde wold expell 6 vnces of water. And likewaies of the syluer: wherof if I had but 2 pounde, & find that it doth expell iij. vnces of water, then might I affirme that 8 li. wold expel 12 vnces, ꝑ is 1 li. weighte. And so is it as good as if the 3 masses wer all of one weight. And thus for this time I wyll make an ende of this other part of Arithmetike.

S. Although I can not sufficientlye thank you for this, yet youre promise made mee to loke for the arte of Extraction of rootes, wherof hytherto I haue learned nothing.

M. I wyl not breake my promise, but intende (God willinge) to perfourme it within these three or foure monethes, yf I perceiue this my pains to bee well taken in the meane season. And you shall not repente the taring for it, for it shalbe increased by the taryinge. And in the meane tyme, you shall take this addition not for the seconde parte of Arithmetike whiche I promised, but for an augmentation of the fyrste parte, vnto whiche I wold haue annexed the Extraction of Rotes, Square and cubik, namely for Examples of statut of Assise of wod, but that in the Secō part I must write of diuers other Rotes, and therefore thoughte it beste to rescrue thos Rootes also with their examples to the sam Second parte.

Sc. Sir although I can not recompenc your goodnesse, yet I shall alwaies doe min indeuour, to occasion you not to repent you benefite on me thus imploied.

Maist. That recompence is sufficiente f your parte.

Finis.

Imprinted at London in Paules churcheyarde at the signe of the Brasen serpent by Reginalde Wolfe.

ANNO DOMINI M. D. LXVI.